UNIT-2
Network Theorems

Syllabus

Unit II -AC Network Theorems (Applications to dependent & independent sources): Pre- Requisites: Concepts of DC Network Theorems, Electrical Sources &Basic circuital law. Superposition theorem, Thevenin's theorem, Norton's theorem, Maximum power transfer theorem, Reciprocity theorem. Millman's theorem, Compensation theorem, Tellegen's Theorem.

Outcome

Analyze the AC and DC circuits using Kirchhoff's law and Network simplification theorems.

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UNIT – 2

Network Theorems

2.1 Superposition Theorem -

Superposition theorem states that in a network containing more than one voltage source or current source, the total voltage or current in any branch of the network is the phasor sum of voltages or currents produced in that branch by each source acting seperately. As each source is considered, all the other sources are replaced by their internal impedances. This theorem is valid only for linear systems.

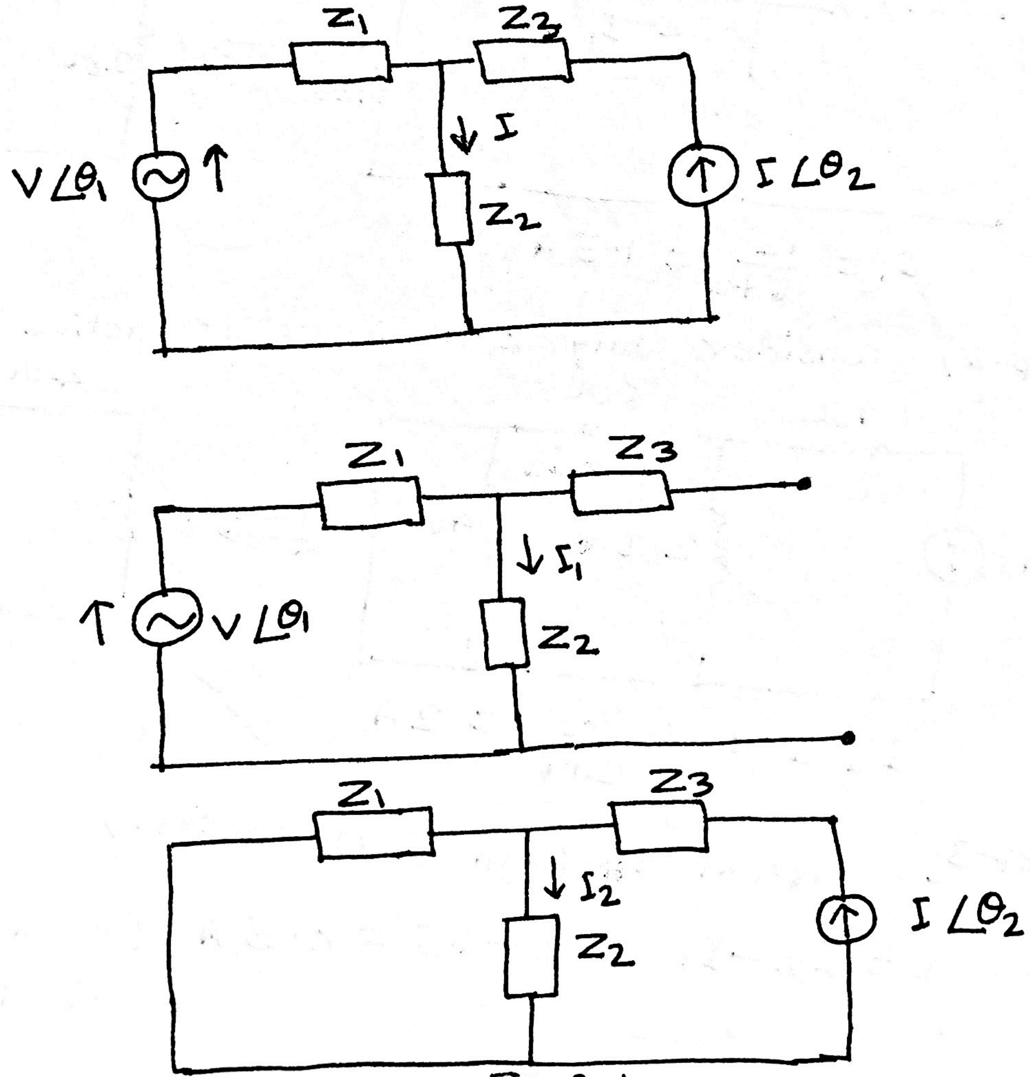


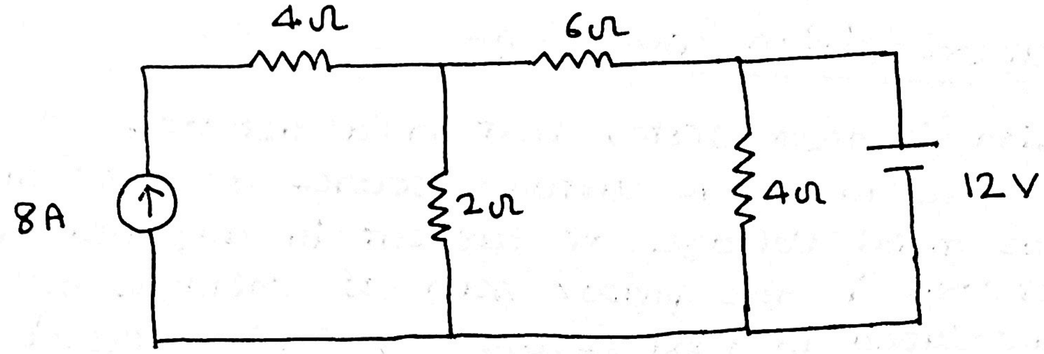
Fig 2.1

$$I_1 = \frac{V\angle\theta_1}{Z_1+Z_2}$$

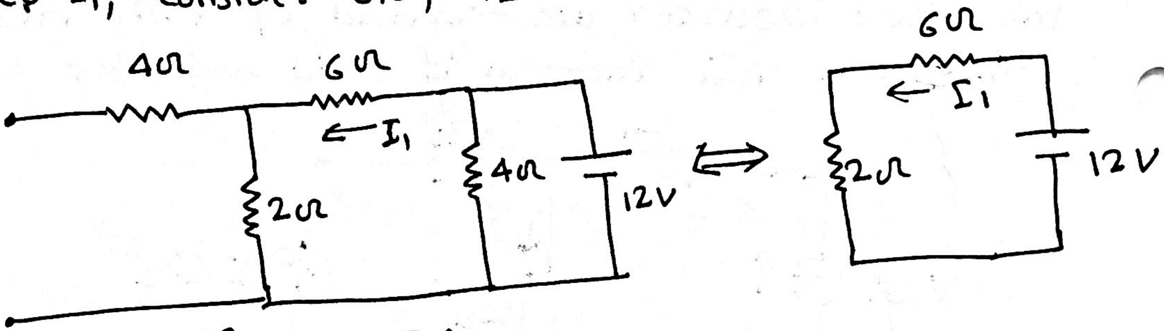
$$I_2 = I\angle\theta_2 \times \frac{Z_1}{Z_1+Z_2}$$

Total current $I = I_1 + I_2$

Example 2.1.1 Determine the current through the 6 Ω shown in fig 2.2 using superposition theorem.

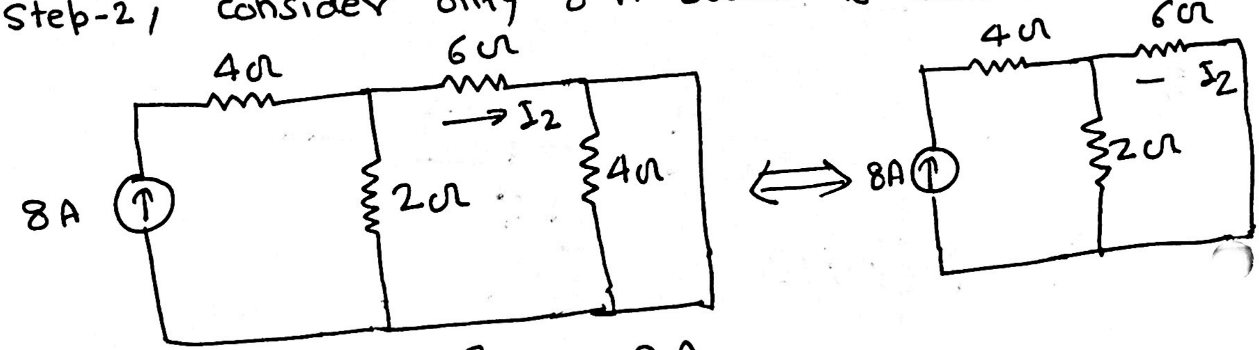


<u>Solution</u> step -1, consider only 12 V source is active



$$I_1 = \frac{12}{6+2} = 1.5\ A$$

Step-2, consider only 8 A source is active

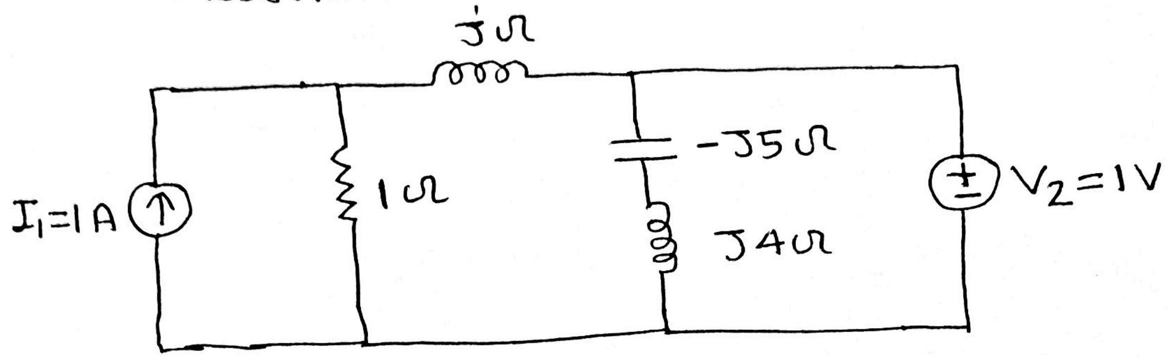


$$I_2 = 8 \times \frac{2}{2+6} = 2\ A$$

<u>Step-3</u> current through 6 Ω resistor

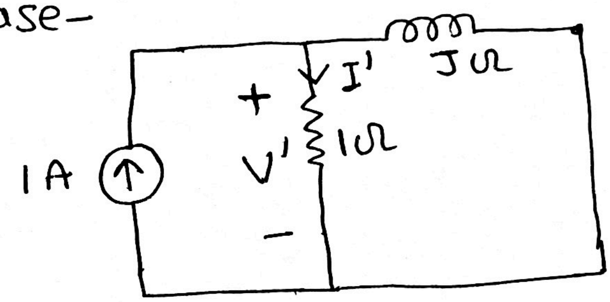
$I = I_2 - I_1 = 2 - 1.5 = 0.5\ A$ (from Left to right)

Problem Calculate voltage V across resistance R in the following circuit using superposition theorem.



Solution - case-1

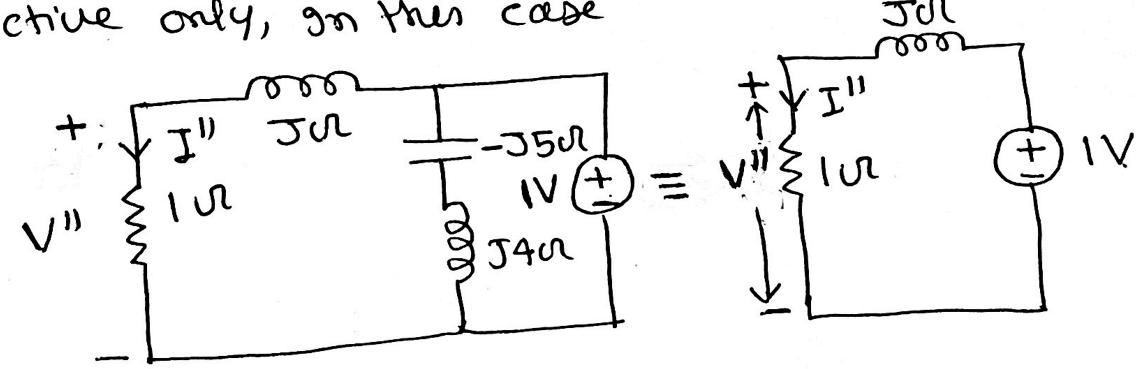
consider only current source $I_1 = 1A$ is active, in this case-



$$I' = 1 \times \frac{j}{j+1}$$

$$V' = I' \times 1 = \frac{j}{j+1}$$

case-2 consider only voltage source $V_2 = 1V$ is active only, in this case



$$I'' = \frac{1}{1+j}$$

$$V'' = I'' \times 1 = \frac{1}{1+j}$$

Hence by superposition theorem voltage across 1Ω resistance when both sources are active is

$$V = V' + V'' = \frac{j}{j+1} + \frac{1}{j+1} = 1\,V$$

Example 2.1.3 Find the current I through the impedance $Z_1 = (4 + j6)\,\Omega$ in the network shown in fig 2.4(a)

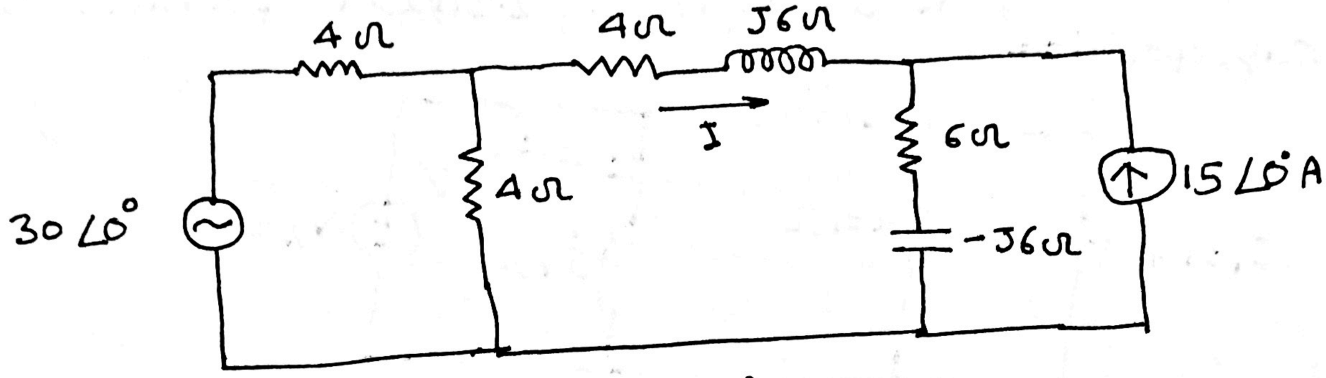
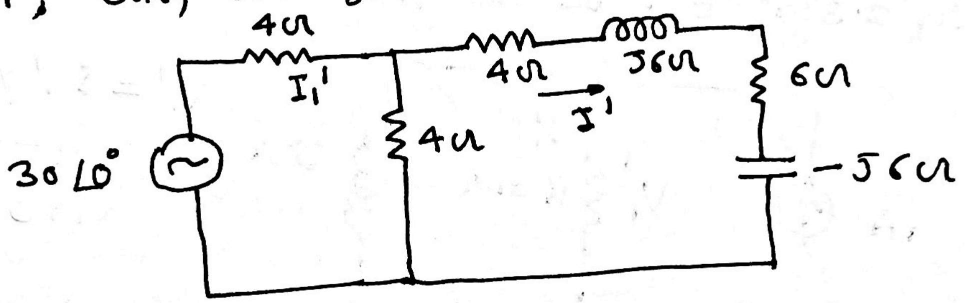


Fig 2.4 (a)

<u>Solution</u> -

Step-1 ; only voltage source $30\angle 0°$ is considered.



$$Z_T = 4 + (4 \,||\, (4 + j6 + 6 - j6))$$

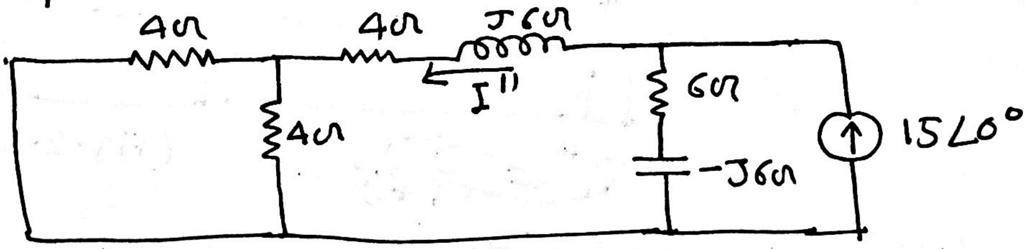
$$= 4 + (4 \,||\, 10) = 4 + 2.857 = 6.857$$

$$I_1' = \frac{30\angle 0°}{Z_T} = \frac{30\angle 0°}{6.857} = 4.375\angle 0°\,A$$

$$I' = I_1' \times \frac{4}{4 + 4 + j6 + 6 - j6} = 4.375\angle 0° \times \frac{4}{14}$$

$$= 1.25\angle 0°\,A = 1.25 + j0$$

Step-2 ; only current source $15\angle 0°\,A$ is considered.



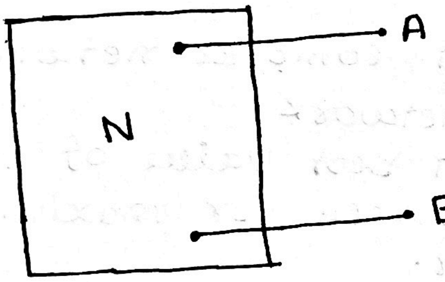
$$I'' = 15\angle 0° \times \frac{6 - j6}{6 - j6 + (4 + j6) + 4 \,||\, 4} = 15\angle 0° \times \frac{6 - j6}{12}$$

$$I'' = (7.5 - j7.5)\,A$$

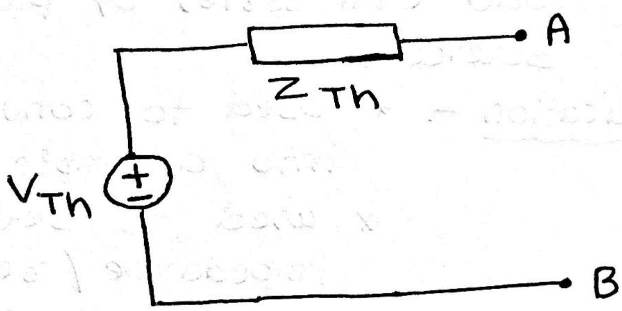
Hence,

$$I = I' - I'' = -6 + j7.5 = 9.6\,\angle 128.7°\,A$$

Thevenin's Theorem - Thevenin's theorem states that " Any two terminal linear network containing energy sources and impedances can be replaced with an equivalent circuit consisting of a voltage source V_{Th} in series with an impedance Z_{Th}." The value of V_{Th} is the open circuit voltage between the terminals of the network and Z_{Th} is the impedance between the terminals of the network with all energy sources set to zero.



An active Network



Thevenin's equivalent

Step-1: calculation of V_{Th} or V_{oc}:
For it remove Z_L (if any) from the terminals and obtain open circuit voltage V_{oc} (or V_{Th}) across the terminals.

Step-2: Calculation of Z_{Th}.
For it remove Z_L (if any) from the terminals and obtain equivalent impedance across the terminals.

Step-3: Draw Thevenin's equivalent circuit by connecting V_{Th} in series with Z_{Th}. Reconnect Z_L across the terminals.

Step-4: Load current is given by

$$I_L = \frac{V_{Th}}{Z_{Th} + Z_L}$$

Voltage across Z_L is given by

$$V_L = I_L \cdot Z_L$$

Calculate I_L and V_L using above formulas.

Limitations of Thevenin's Theorem -

(1) This theorem is not applicable to the networks consisting unilateral elements, e.g. diode, transistor etc.

(2) Not applicable to the network consisting non-linear elements, e.g. diode, transistor etc.

(3) Not applicable to the networks consisting magnetic coupling.

(4) Not applicable to the circuits consisting of load (in series or parallel) with dependent sources.

Application - * Used to convert complex network into a simple network
* used to select best value of load impedance / resistance for maximum power transfer.

Procedure to obtain Z_{Th}:- The procedure to obtain Z_{Th} depends upon the nature of energy sources present in the network. There may be three cases-

(1) Network consisting independent energy source.
(2) Network consisting both independent and dependent energy sources.
(3) Network consisting dependent energy sources only,

(1) Z_{Th} for Networks consisting independent energy sources — In this case, all energy sources are replaced by their internal impedances.

* Ideal voltage source is replaced by short circuit
* Ideal current source is replaced by open circuit.

(2) Z_{Th} for network consisting both independent and dependent energy sources –

In this case

$$Z_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{V_{oc}}{I_{sc}}$$

V_{Th} = Thevenin's voltage

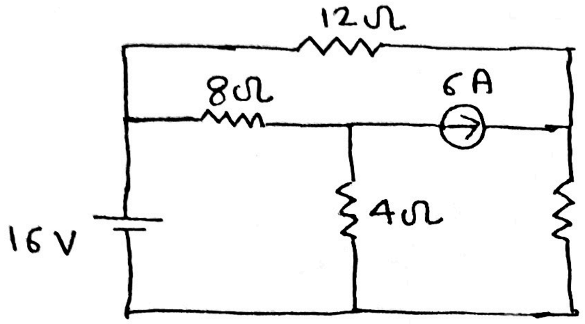
I_{sc} = Short circuit current.

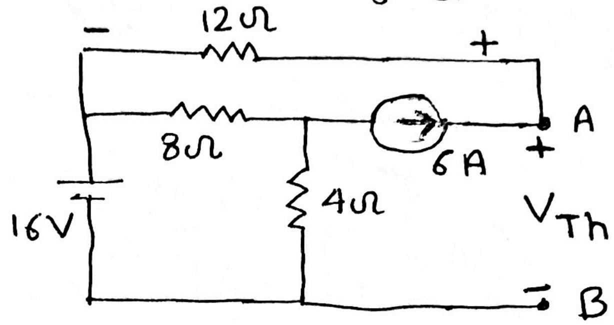
(3) Z_{Th} for the network consisting only dependent energy sources –

In this case, connect a voltage source of voltage V at the terminals. If current I is flow due to the application of V, then Z_{Th} is given as –

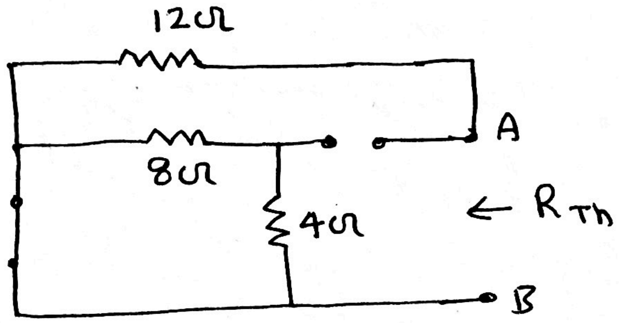
$$Z_{Th} = \frac{V}{I}$$

Problem - Using Thevenin's Theorem, find the current flowing through 10 Ω resistor of the network shown in fig - a

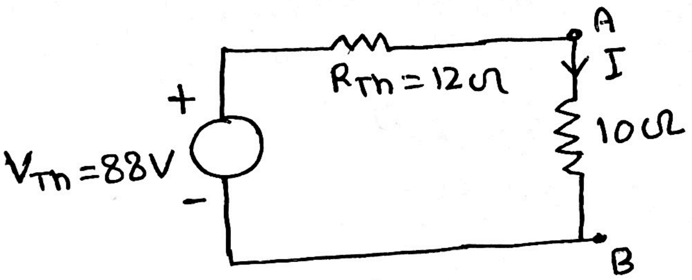




$$V_{Th} = 12 \times 6 + 16 = 88\,V$$



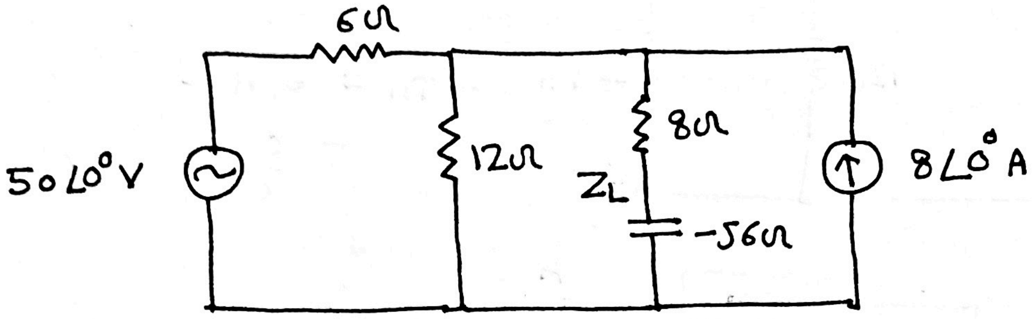
$$R_{Th} = 12\,\Omega$$



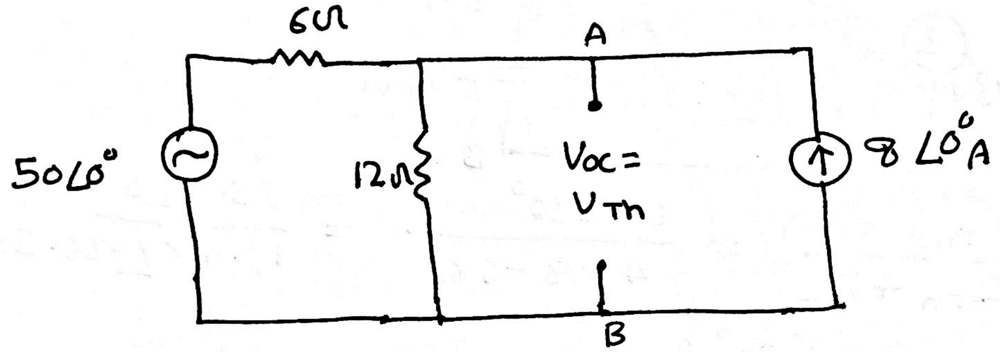
$$I = \frac{V_{Th}}{R_{Th} + 10}$$

$$= \frac{88}{12 + 10} = 4A$$

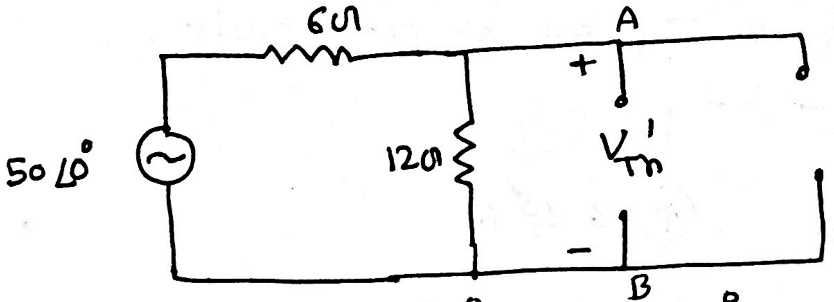
Example Find the current through Z_L using Thevenin's theorem. verify the result using Norton's theorem



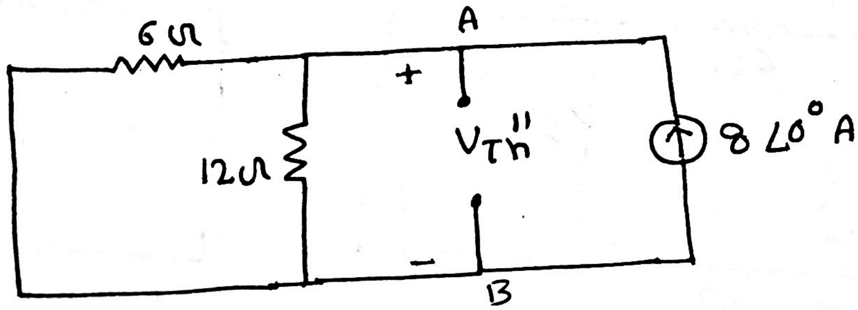
Solution



Step-1 To obtain V_{Th} we use superposition theorem.



$$V_{th}' = \frac{50\angle 0^\circ}{(6+12)} \times 12 = \frac{100\angle 0^\circ}{3} V = 33.3\angle 0^\circ V$$

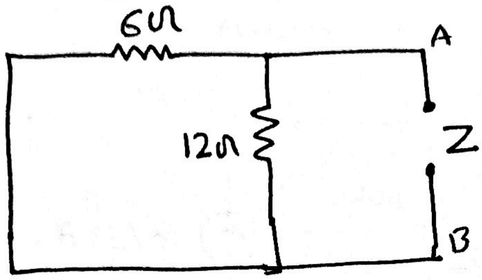


$$V_{th}'' = 8\angle 0^\circ \times \frac{6}{(6+12)} \times 12 = \frac{8\angle 0^\circ \times 6 \times 12}{18}$$

$$= 32.0\angle 0^\circ V$$

$$V_{Th} = V_{Th}' + V_{Th}'' = 65.3\angle 0^\circ$$

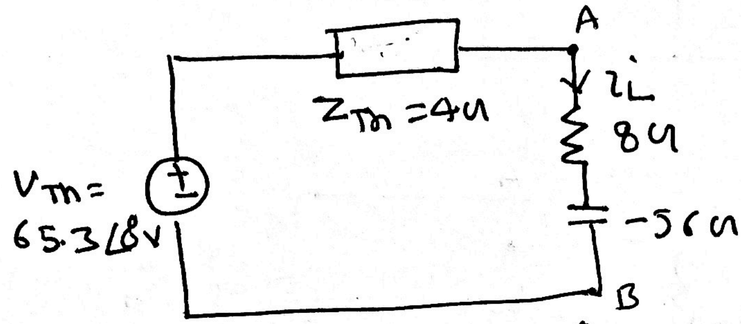
Step-2, To obtain Z_{Th}



$$Z_{Th} = 6 \parallel 12$$

$$= \frac{6 \times 12}{6+12} = 4\,\Omega$$

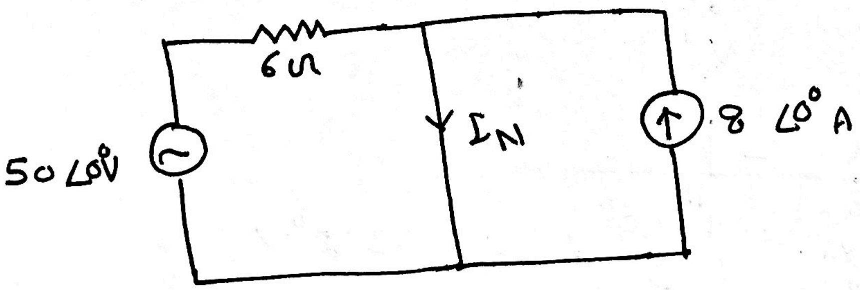
Step-3

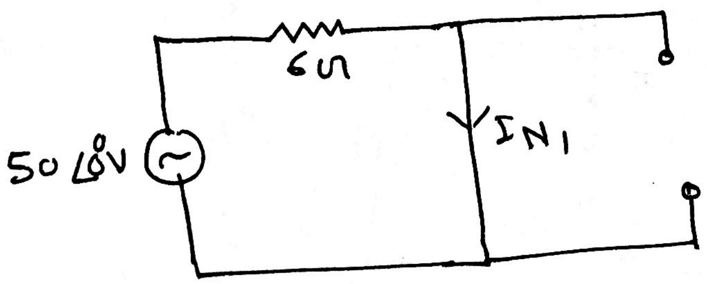


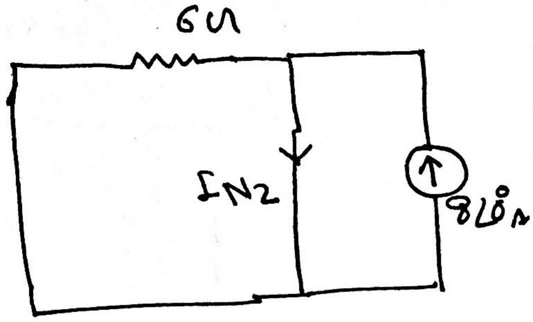
$$i_L = \frac{V_{Th}}{Z_{Th} + Z_L} = \frac{65.3\angle 0°}{4+8-j6} = \frac{65.3\angle 0°}{13.416\angle -26.56°}$$

$$= 4.87\angle 26.56°\ \text{A}$$

To obtain I_N, we replace Z_L by short-circuit.







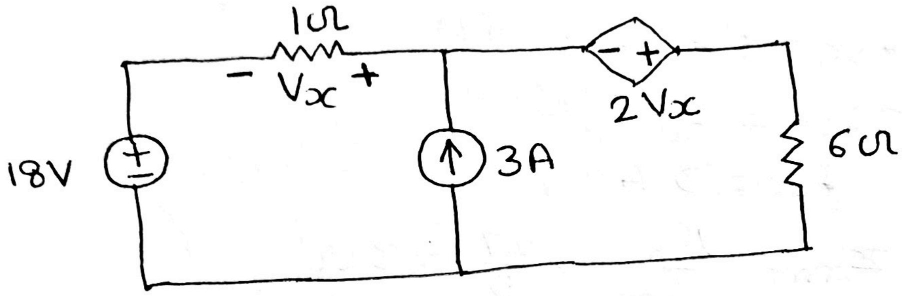
$$I_{N1} = \frac{50\angle 0°}{6}$$

$$I_{N2} = 8\angle 0°$$

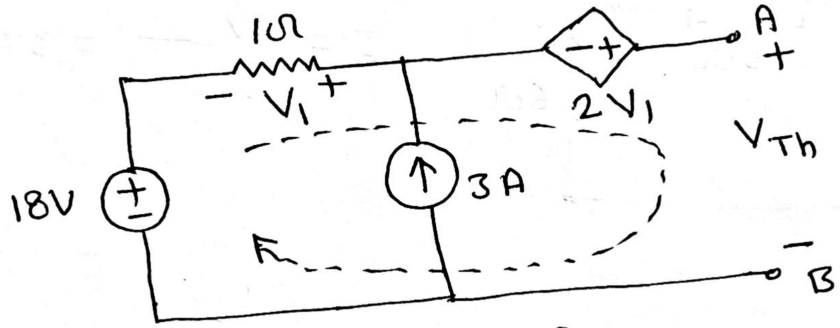
$$I_N = I_{N1} + I_{N2} = \frac{50}{6}\angle 0° + 8\angle 0°$$

$$= 16.33\angle 0°$$

Prob Calculate the current through 6 Ω resistor in the circuit shown in following fig-



Solution step-1 (calculation of V_{Th})



$$-1\times 3 - 2V_1 + V_{Th} - 18 = 0$$

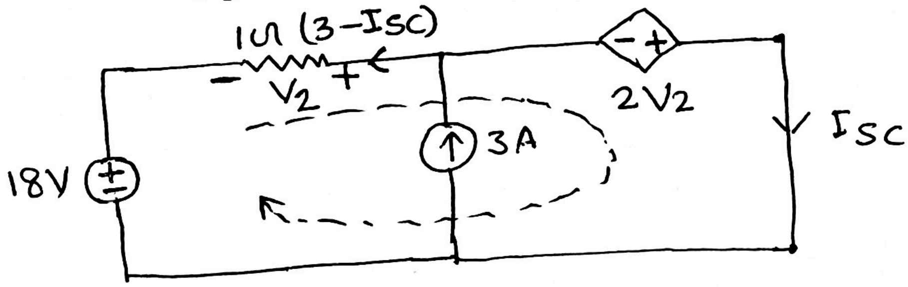
$$V_{Th} = 2V_1 + 21 \quad \text{---- (1)}$$

where $V_1 = 1\times 3 = 3\,V$

$$V_{Th} = 2\times 3 + 21 = 27\,V$$

Step-2 calculation of Z_{Th}

* we know that when dependent and independent both sources are present in circuit then Z_{Th} is given by $Z_{Th} = \dfrac{V_{oc}}{I_{sc}} = \dfrac{V_{Th}}{I_{sc}}$



By KVL,

$$-1\times(3 - I_{sc}) - 2V_2 - 18 = 0$$

$$-3 + I_{sc} - 2V_2 - 18 = 0$$

$$I_{sc} = 2V_2 + 21 \quad \text{---- (2)}$$

where $V_2 = 1 \times (3 - I_{sc})$

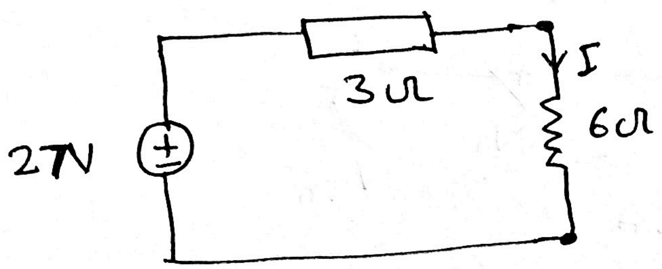
hence $I_{sc} = 2(3 - I_{sc}) + 21$

$I_{sc} = 6 - 2I_{sc} + 21$

or $3I_{sc} = 27$

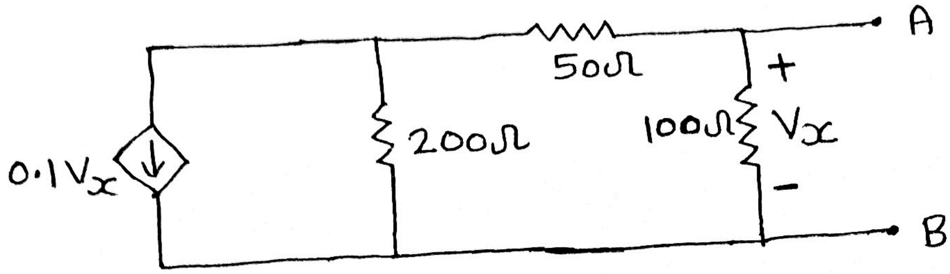
$I_{sc} = 9\ A$

Therefore $Z_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{27}{9} = 3\,\Omega$



$I = \frac{27}{3+6} = 3\ A$

Prob Determine Thevenin's parameters of the network shown in following figure across terminals A–B.



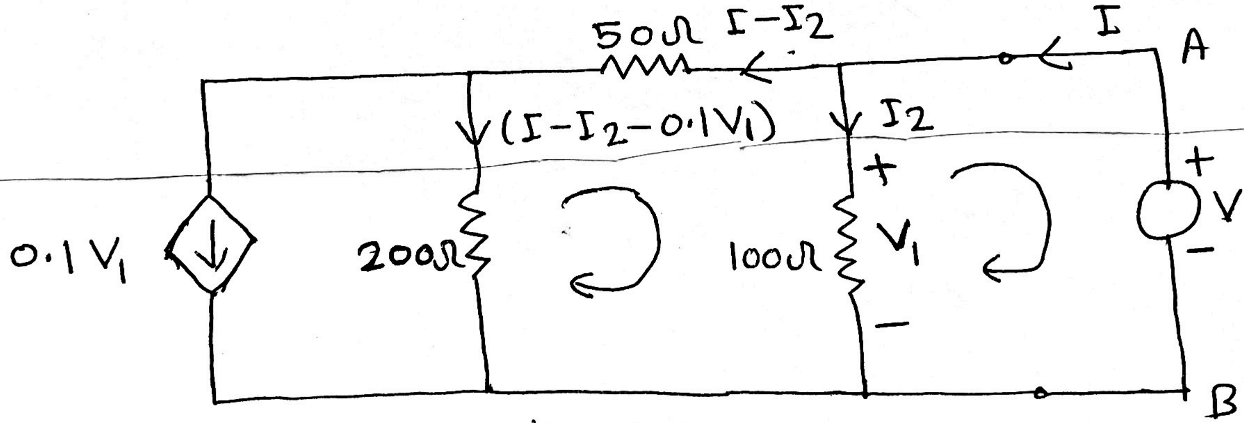
Solution – step-1: Calculation of V_{Th}.

since circuit has only dependent energy source, hence $\boxed{V_{th} = 0}$

step-2: calculation of Z_{Th}.

when network has only dependent energy source, then a voltage source V is applied and let current I flows due to V, Then Z_{Th} is given by

$$Z_{Th} = \frac{V}{I}$$



By KVL, we can write loop equations–

$$-50(I-I_2) + V_1 - 200(I - I_2 - 0.1V_1) = 0$$

or $-50I + 50I_2 + V_1 - 200I + 200I_2 + 20V_1 = 0$

or $-250I + 250I_2 + 21V_1 = 0$ —— (1)

and $V_1 = 100I_2$

$V_1 = V$

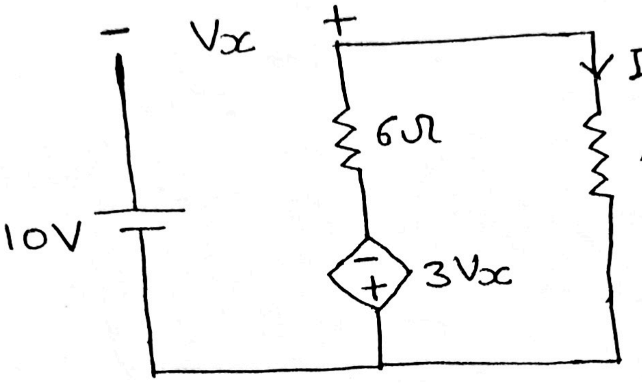
$$I_2 = \frac{V_1}{100} = \frac{V}{100}$$

hence $-250I + 250\frac{V}{100} + 21V = 0$

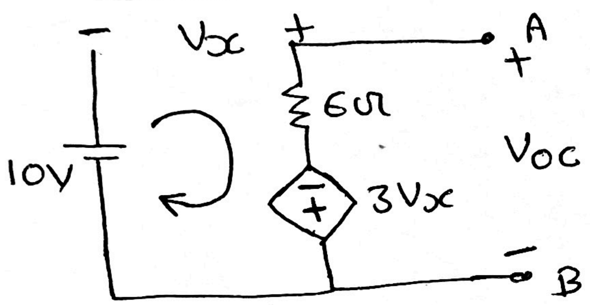
$-250I + 23.5V = 0 \Rightarrow \frac{V}{I} = \frac{250}{23.5} = 10.63\Omega$

or $\boxed{Z_{Th} = \frac{V}{I} = 10.63\Omega}$

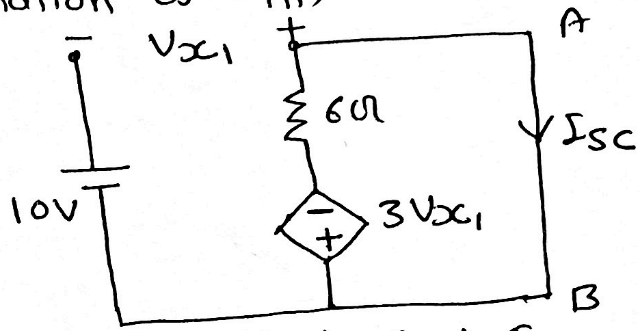
Problem For the circuit shown in following figure, find the current I in the 4 Ω resistor



Solution Step-1 (Determination of V_{Th})



circuit to find V_{Th}



circuit to find I_{sc}

$$-V_x - 3V_x - 10 = 0$$

or $\quad 4V_x = -10$

$$V_x = -\frac{10}{4} = -2.5V$$

$$V_{Th} = V_{oc} = -3V_x = -3 \times (-2.5) = 7.5V$$

Step-2 Determination of I_{sc}

$- V_{x1} - 10 = 0 \quad$ or $\quad V_{x1} = -10V$

and $\quad 3V_{x1} + 6I_{sc} = 0$

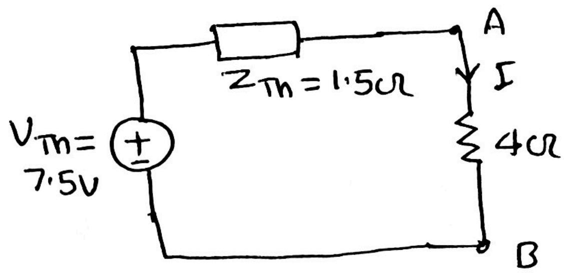
or $\quad I_{sc} = -\frac{3}{6}V_{x1} = -\frac{1}{2} \times (-10) = 5V$

$$Z_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{7.5}{5} = 1.5Ω$$

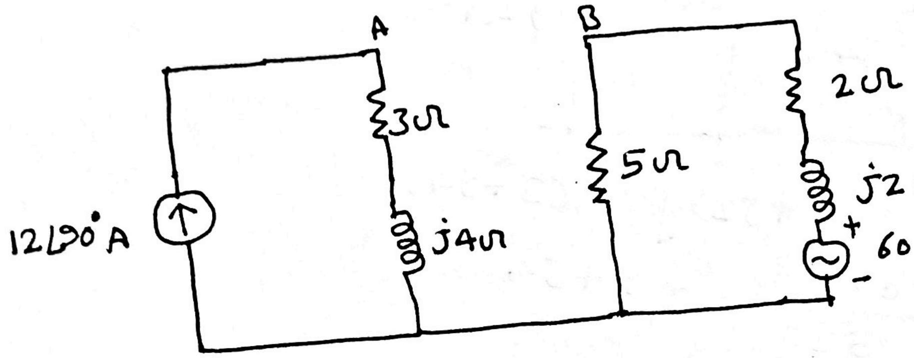
Step-3 Determination of current in 4Ω resistor

$$I = \frac{V_{Th}}{Z_{Th} + 4} = \frac{7.5}{1.5 + 4}$$

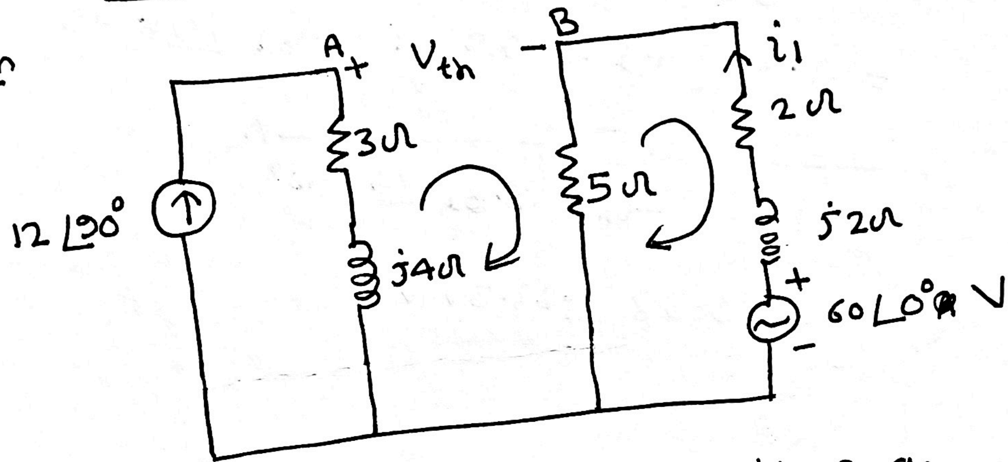
$$I = 1.36\ A$$



Example — Determine Thevenin's equivalent circuit parameters across terminals A–B for the circuit shown in fig –



Solution



By KVL, we can write loop equations as

$-(3+j4)\times 12\angle 90 + V_{th} + 5i_1' = 0$

or $\quad V_{th} = (3+j4)\times 12\angle 90 - 5i_1' \quad$ —— (1)

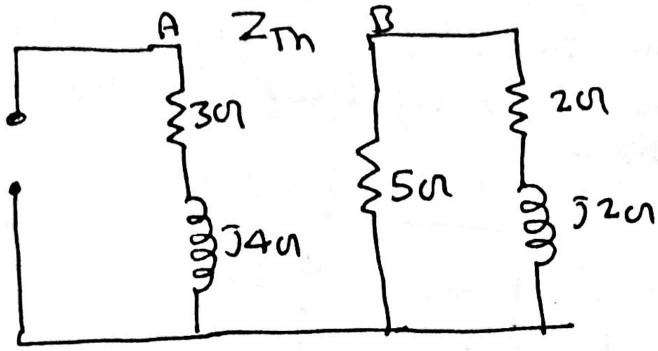
and $\quad -(2+j2)i_1' + 60 - 5i_1' = 0$

or $\quad -(7+j2)i_1' + 60 = 0$ —— (2)

or $\quad i_1' = \dfrac{60}{7+j2}$

$V_{Th} = (3+j4)\times 12\angle 90 - 5\times \dfrac{60}{(7+j4)}$

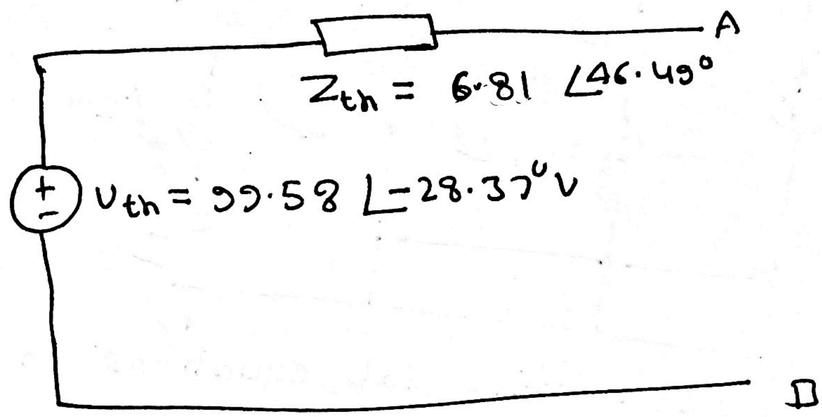
$= 99.58 \angle -28.37°$ Volt



$$Z_{Th} = \left[5 \parallel (2+j2)\right] + (3+j4)$$

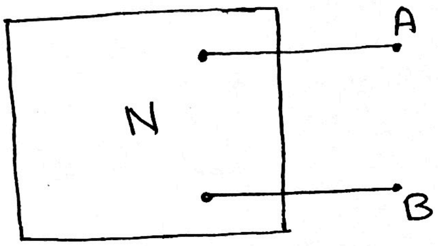
$$= \frac{90+j50}{53} + 3 + j4$$

$$= 4.69 + j\,4.49 = 6.81 \angle 46.49$$

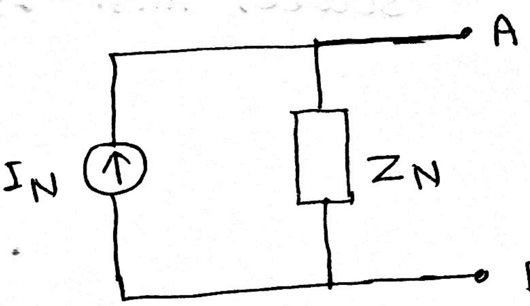


Norton's Theorem - Norton's theorem states that "Any two terminal linear network containing energy sources and impedances can be replaced by an equivalent circuit consisting of a current source I_N parallel with an internal impedance Z_N".

The value of I_N is the short circuit current between the terminals of the network and Z_N is the impedance measured between the terminals with all energy sources set to zero.



An active network



Norton's Equivalent circuit

Procedure to apply Norton's Theorem -

Step-1 - For calculation of I_N or I_{sc} -

Remove Z_L from the network and short the teminals. Find I_{sc} or I_N.

Step-2 - Find Z_N using the same procedure as Z_{Th}.

Step-3 - Draw Norton's equivalent circuit by connecting I_N or I_{sc} in parallel with Z_N. Reconnect Z_L across the terminals.

Step-4 - Load current is given by -

$$I_L = I_N \frac{Z_N}{Z_N + Z_L}$$

and voltage across the load Z_L

$$V_L = I_L Z_L$$

Application of Norton's Theorem -

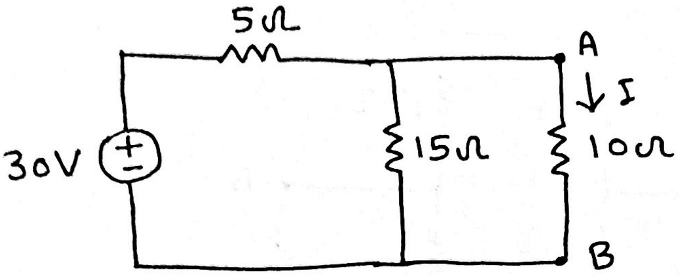
It can be used to convert a complicated network into a simple network to find voltage, current and power delivered to the load.

Limitations of Norton's Theorem -

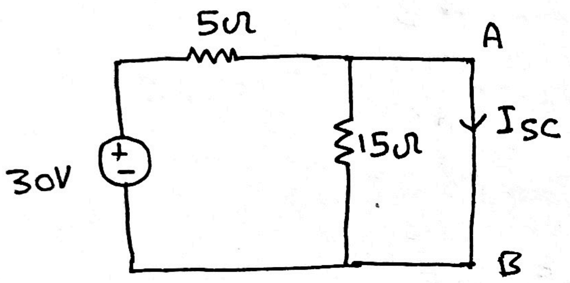
The limitations of Norton's theorem are same as Thevenin's theorem.

Note - If a network has only dependent energy sources then $I_{sc} = 0$ always.

Problem : Determine the current through 10 Ω resistance shown in figure using Norton's theorem.

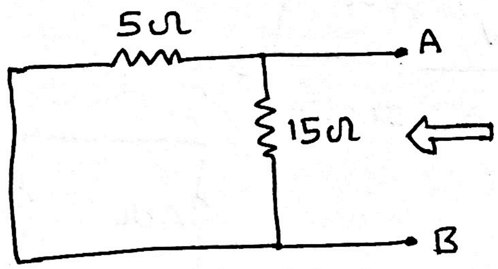


Solution - Step-1 Determination of I_{SC},



$$I_{SC} = \frac{30}{5} = 6\,A$$

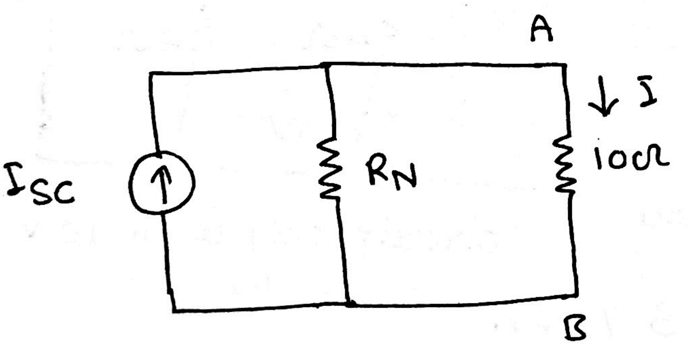
Step-2 - Determination of R_N,



$$R_N = 5 \| 15 = \frac{5 \times 15}{5+15}$$

$$= 3.75\,\Omega$$

Step-3 Determination of I,

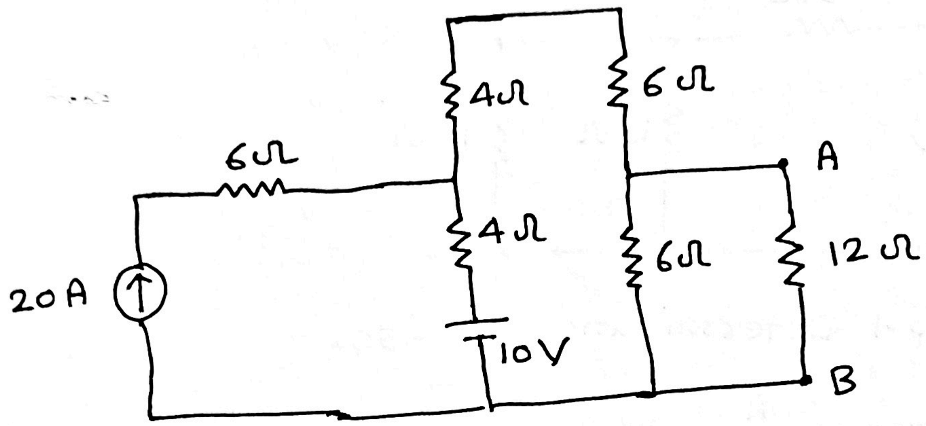


$$I = \frac{I_{SC} \times R_N}{R_N + 10}$$

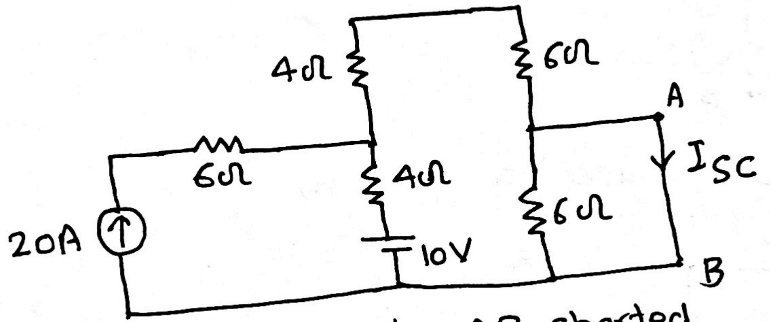
$$= \frac{6 \times 3.75}{3.75 + 10}$$

$$I = 1.636\,A$$

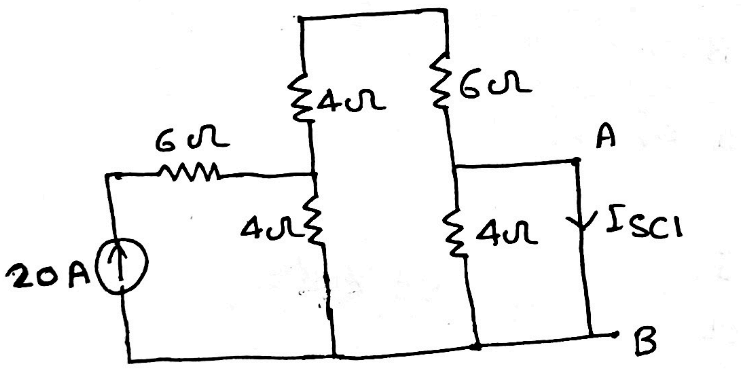
Problem - Determine the current in the 12Ω resistor shown in fig using Norton's theorem.



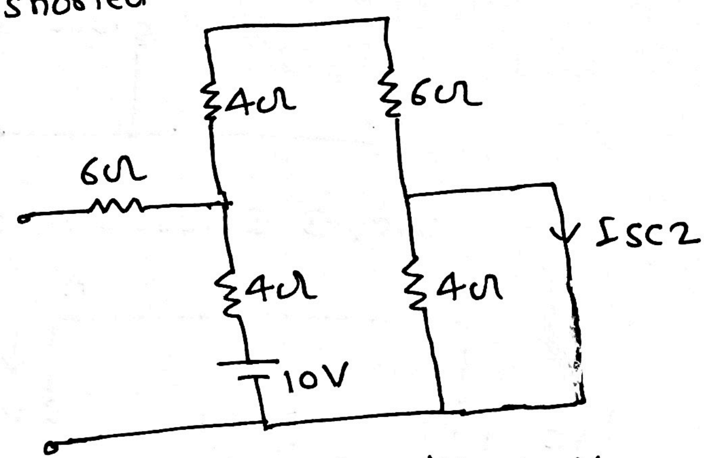
Solution - Step 1: Determination of I_{sc} or I_N.



circuit with AB shorted



Circuit only with 20 A source



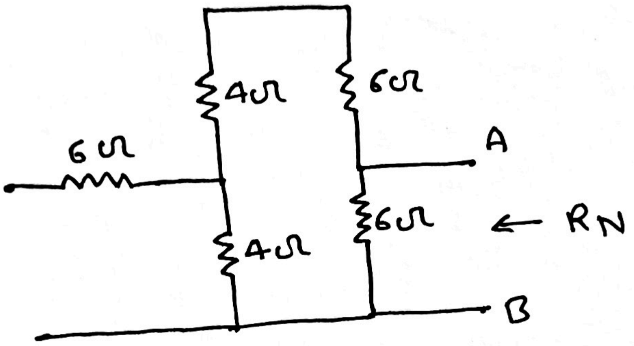
circuit only with 10 V source

$$I_{sc1} = 20 \times \frac{4}{4+6+4} = 5.714\ A$$

$$I_{sc2} = \frac{10}{4+4+6} = 0.714\ A$$

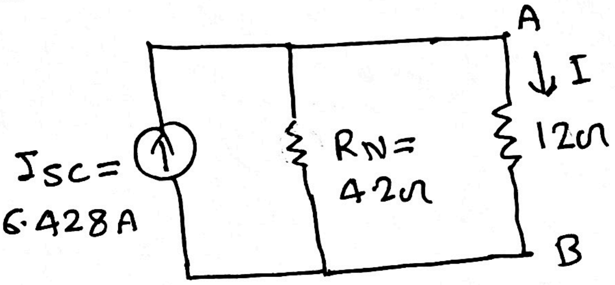
$$I_{sc} = I_{sc1} + I_{sc2} = 5.714 + 0.714 = 6.428\ A$$

Step-2 Determination of R_N or R_{Th}



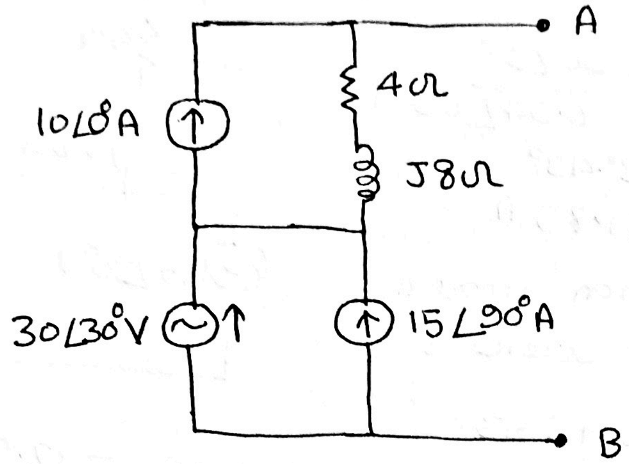
$$R_N = (6+4+4) \parallel 6$$
$$= 14 \parallel 6$$
$$= \frac{14 \times 6}{14+6} = 4.2\,\Omega$$

Step-3 Determination of current in 12Ω resistor.



$$I = I_{sc} \times \frac{R_N}{R_N + 12}$$
$$= 6.428 \times \frac{4.2}{4.2 + 12}$$
$$= 1.666\,A$$

Problem Find the Norton's equivalent circuit for the circuit shown in following figure.

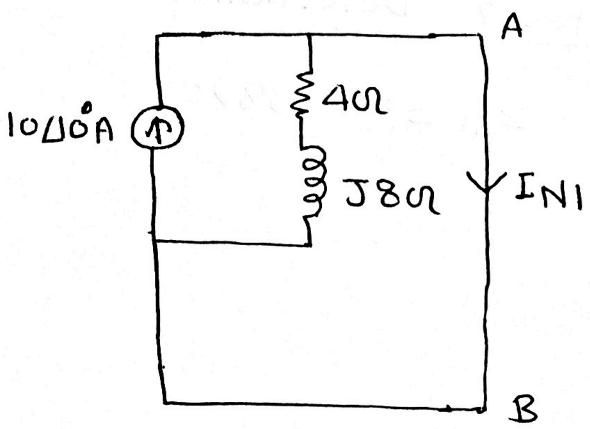


Step-1 Determination of I_N

(a) consider only 10 ∠0° A source is active

In this case

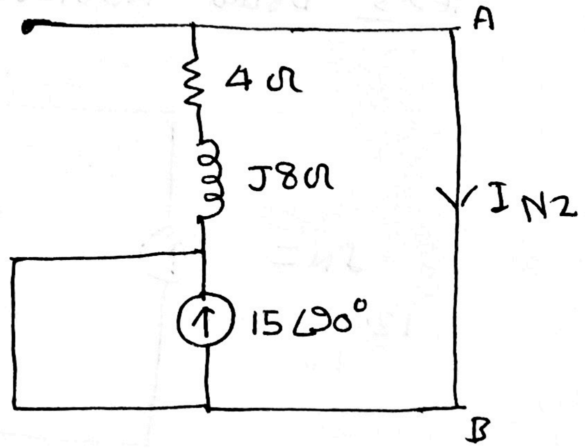
$I_{N1} = 10 \angle 0°$



(b) consider only 15 ∠90° current source is active

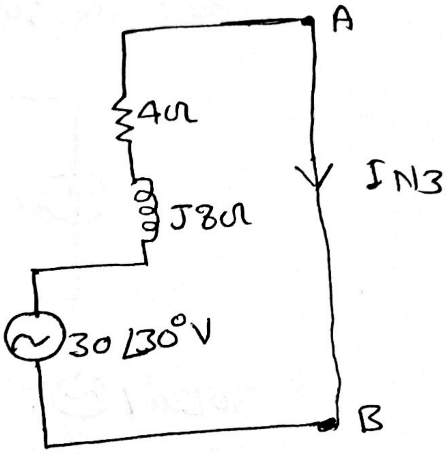
In this case

$I_{N2} = 0$



(c) consider only $30\angle 30^\circ$ V source is active.

In this case



$$I_{N3} = \frac{30\angle 30^\circ}{4+J8} = \frac{30\angle 30^\circ}{8.94\angle 63.43}$$

$$= 3.35\angle -33.43^\circ$$

$$= 2.8 - J1.85 \text{ A}$$

So, using super-position theorem,

Norton's current source is

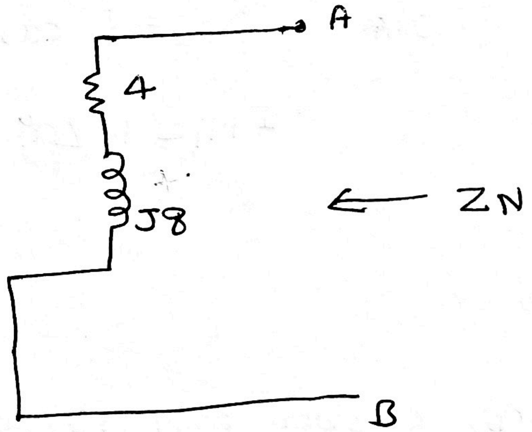
$$I_N = I_{N1} + I_{N2} + I_{N3}$$

$$= (10+J0) + 0 + 2.8 - J1.85 = 12.8 - J1.85$$

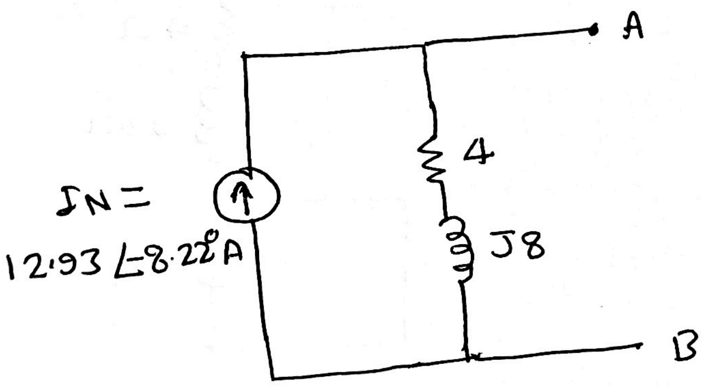
$$= 12.93\angle -8.22^\circ \text{ A}$$

Step-2 Determination of Z_N.

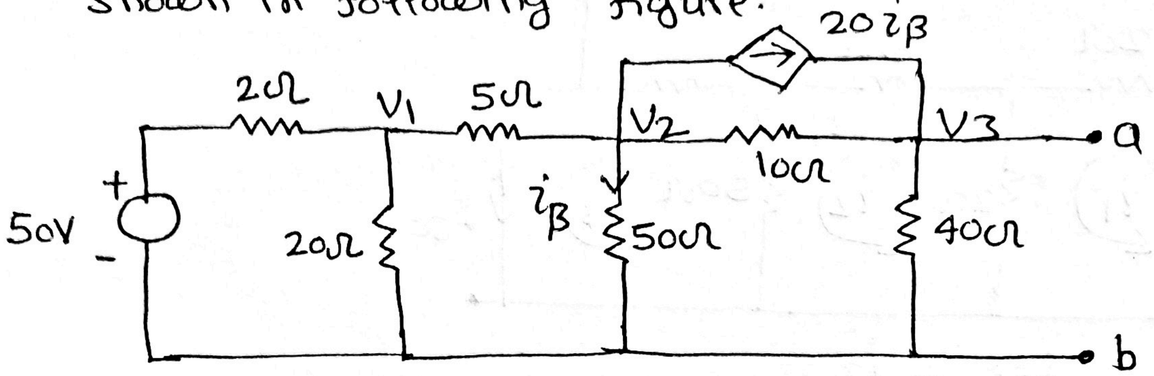
$$Z_N = (4+J8)\,\Omega$$



Step-3 Draw Norton's equivalent circuit



Prob | Find equivalent current generator for the circuit shown in following figure.



$I_{sc} = ?$

$R_N = \dfrac{V_{oc}}{I_{sc}}$

$V_{oc} = ?$

Solution | step-1 Determination V_{oc}

$V_{oc} = V_3$

$$\frac{V_1 - 50}{2} + \frac{V_1}{20} + \frac{V_1 - V_2}{5} = 0$$

or $\quad 15 V_1 - 4 V_2 = 500 \quad -(1)$

$$\frac{V_2 - V_1}{5} + \frac{V_2}{50} + \frac{V_2 - V_3}{10} + 20 i_\beta = 0$$

or $\quad -10 V_1 + 36 V_2 - 5 V_3 = 0 \quad -(2)$

$$\frac{V_3}{40} + \frac{V_3 - V_2}{10} - 20 i_\beta = 0$$

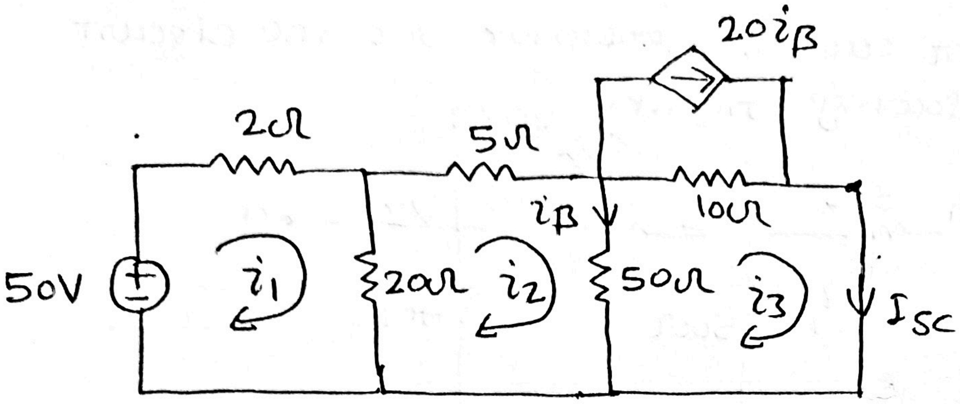
or $\quad -3.4 V_2 + 5 V_3 = 0 \quad -(3)$

Solving eq^n (1), (2) and 3, we get

$V_1 = 36.4V$, $V_2 = 11.5V$, $V_3 = 10.12V$

$V_{oc} = V_3 = 10.12V$

Step-2 Determination of I_{sc},



$I_{sc} = i_3$

- KVL equations for the above circuit can be written as -

$$-50 + 2i_1 + 20(i_1 - i_2) = 0$$

or $$11 i_1 - 10 i_2 = 25 \quad \text{---(4)}$$

$$5i_2 + 50(i_2 - i_3) + 20(i_2 - i_1) = 0$$

or $$-4i_1 + 15i_2 - 10i_3 = 0 \quad \text{---(\ )}$$

$$10(i_3 - 20 i_\beta) + 50(i_3 - i_2) = 0$$

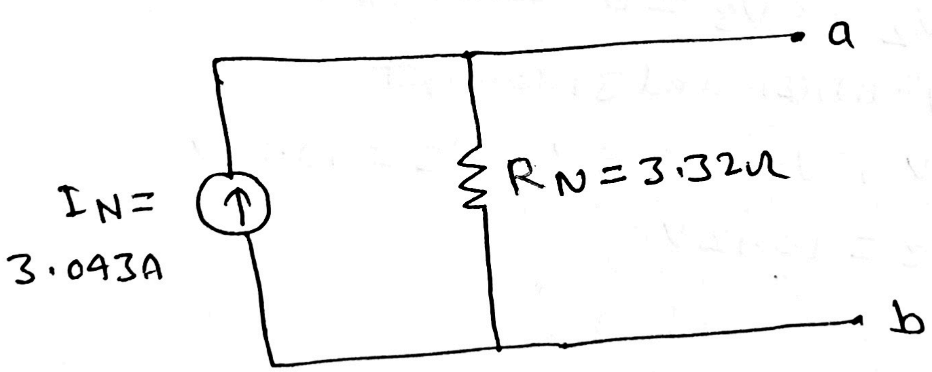
or $$7i_2 - 8i_3 = 0 \quad \text{---(6)}$$

solving eqn 4, 5 and 6, we get

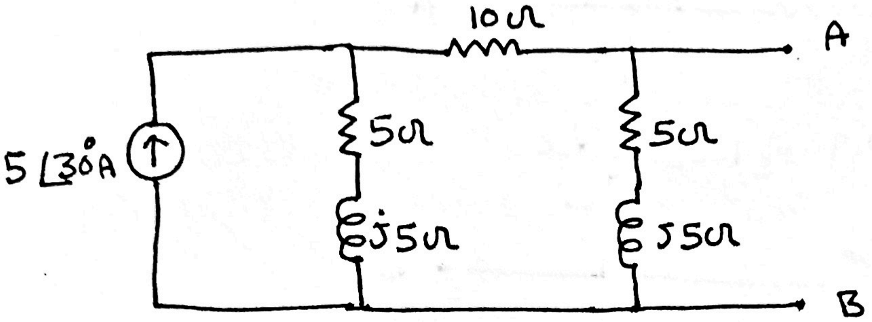
$i_1 = 5.434A$, $i_2 = 3.47A$, $i_3 = 3.043A$

$I_{sc} = i_3 = 3.043A$

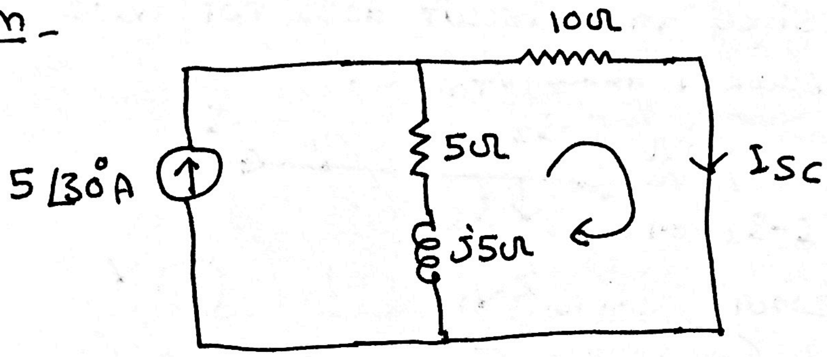
$$R_N = \frac{V_{oc}}{I_{sc}} = \frac{10.12}{3.043} = 3.325\Omega$$



Example Find Norton's parameters across terminals A–B for the circuit shown in fig –

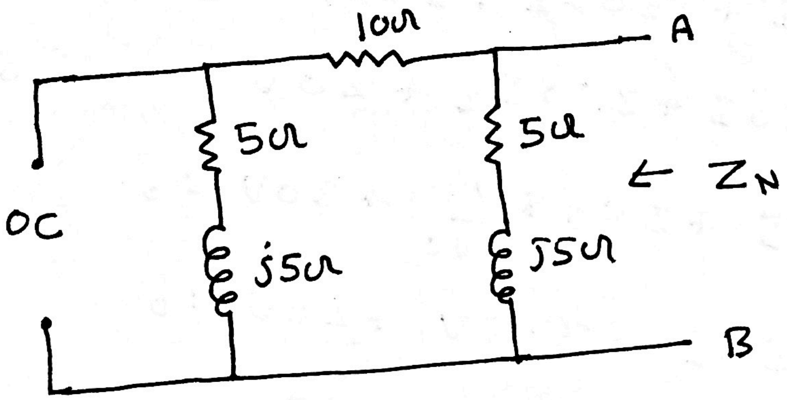


Solution –



$$I_{sc} = \frac{5\angle 30^\circ \times (5+j5)}{10+5+j5} = \frac{5\angle 30^\circ \times (5+j5)}{(15+j5)}$$

$$= \frac{5\angle 30^\circ \times 5\sqrt{2}\angle 45^\circ}{15.8\angle 18.43^\circ} = 2.236\angle 56.57^\circ\ A$$



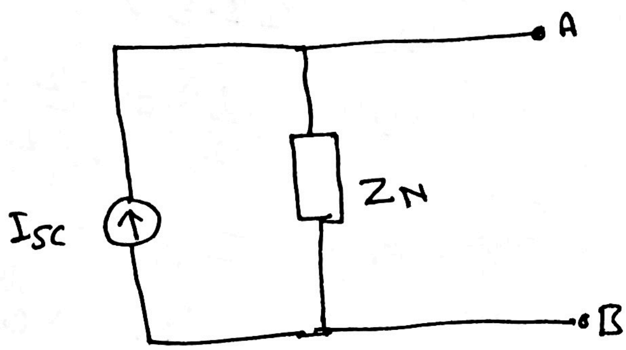
$$Z_N = (10+5+j5) \,||\, (5+j5) = \frac{50+j100}{20+j10}$$

$$= 4.99\angle 36.87^\circ$$

Hence Norton's parameters are –

$I_N = I_{sc} = 2.236\angle 56.57^\circ\ A$

$Z_N = 4.99\angle 36.87^\circ$



Reciprocity Theorem – Reciprocity theorem states that "In a linear, bilateral, active, single source network, the ratio of excitation to response remains same when the positions of excitation and response are interchanged." This theorem is valid for circuits containing only one independent source and no dependent sources.

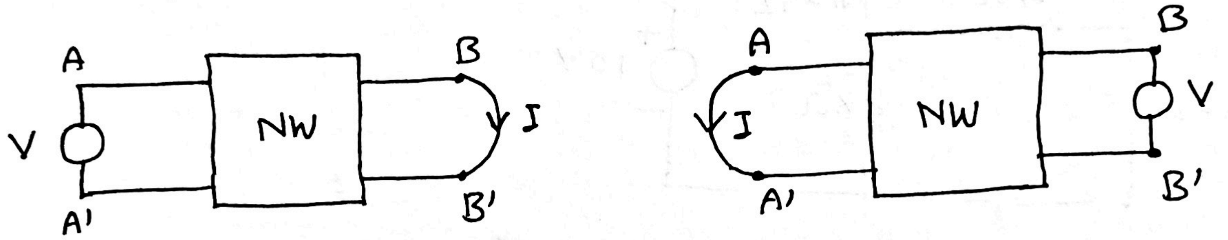


Fig - a

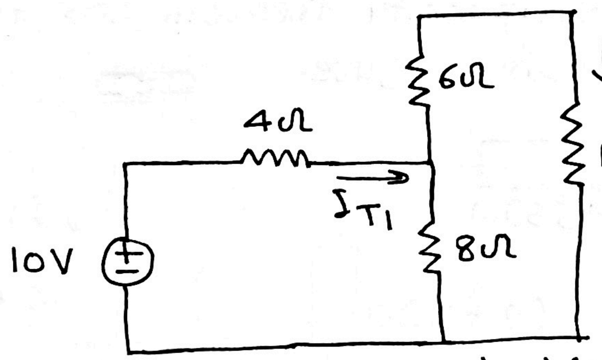
Procedure to apply Reciprocity Theorem –

(1) Identify the branches between which reciprocity is to be established.

(2) Find the current in the branch when excitation and response are not interchanged.

(3) Find the current in the branch when excitation and response are interchanged.

Problem – Verify the reciprocity theorem for the circuit shown in following figure.



circuit of problem (orginal)

Solution – Excitation and Response in original Position –

$$R_T = 4 + [8 \| (6+12)] = 9.538\,\Omega$$

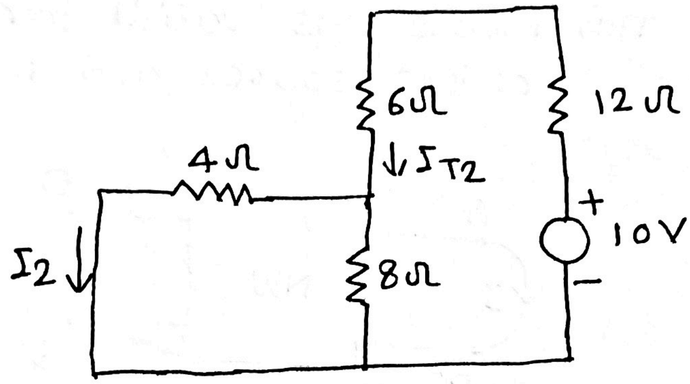
$$I_{T_1} = \frac{10}{9.538} = 1.048\text{ A}$$

$$I_1 = I_{T_1} \times \frac{8}{8+6+12} = 1.048 \times \frac{8}{26} = 0.3226\text{ A}$$

Ratio of Excitation (Voltage) to Response (current)

$$= \frac{10}{0.3226} = 31$$

<u>Positions of Excitation and Response interchanged</u> -



$$R_T = 12 + 6 + (4 \| 8) = 20.66\,\Omega$$

$$I_{T2} = \frac{10}{20.66} = 0.4838\ A$$

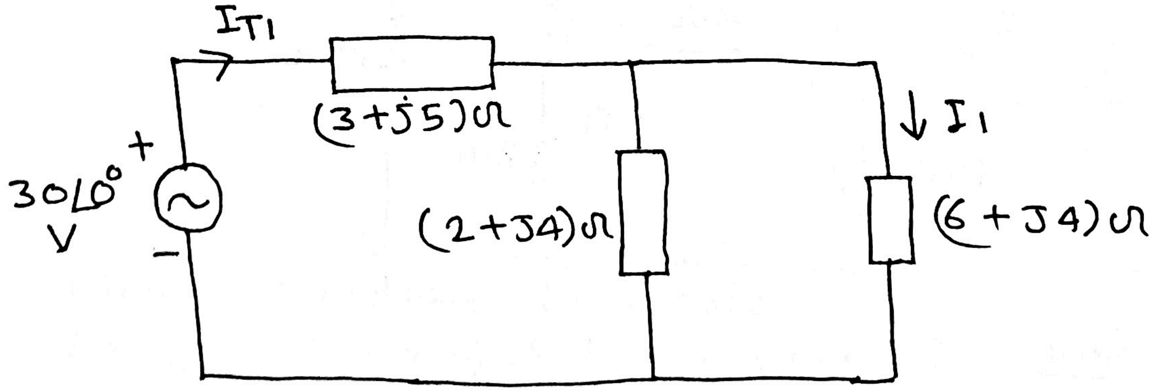
$$I_2 = I_{T2} \times \frac{8}{8+4} = 0.4838 \times \frac{8}{12} = 0.3226A$$

Ratio of excitation (voltage) to response (current)

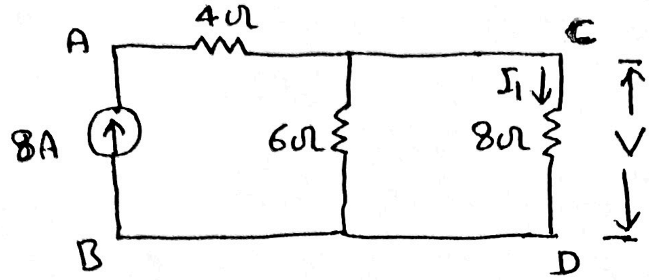
$$= \frac{10}{0.3226} = 31$$

* In both cases, the ratio of voltage to current is 31, hence reciprocity theorem is verified.

<u>Problem</u> - Verify the reciprocity theorem for the network shown in following figure.



Problem - Verify reciprocity theorem in the circuit shown in following figure -



Solution - Step-1: Excitation and response in original position.

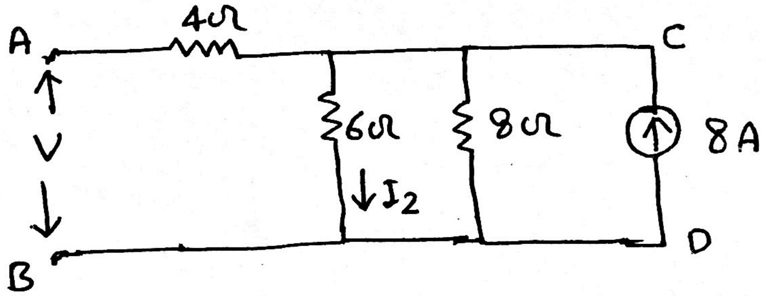
$$I_1 = \frac{6}{8+6} \times 8 = \frac{48}{14} = 3.428\ A$$

$$V = I_1 \times 8 = 3.428 \times 8 = 27.43\ V$$

Ratio of excitation (current) to response (Voltage)

$$= \frac{8}{27.43} = 0.29$$

Step-2: Positions of Excitation and response interchanged.



$$I_2 = 8 \times \frac{8}{6+8} = 4.571\ A$$

$$V = 6 \times I_2 = 6 \times 4.571\ A$$
$$= 27.43\ V$$

Ratio of excitation (current) to Response (Voltage) $= \frac{8}{27.43} = 0.29$

In both case ratio of excitation to response is same, hence reciprocity theorem is verified

TELLEGEN'S THEOREM - Tellegen's theorem states that in a given network, the algebraic sum of powers delivered by all sources is equal to the algebraic sum of powers absorbed by all elements.

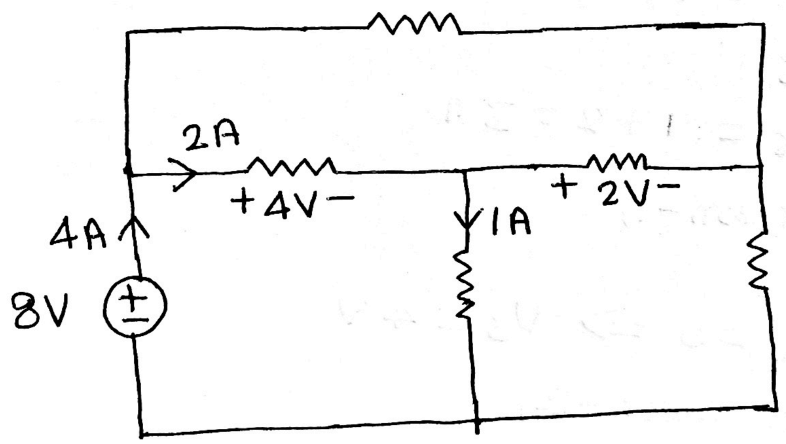
Mathematically,

$$\sum_{K=1}^{n} v_K(t)\, i_K(t) = 0 \quad (\text{for all value of } t)$$

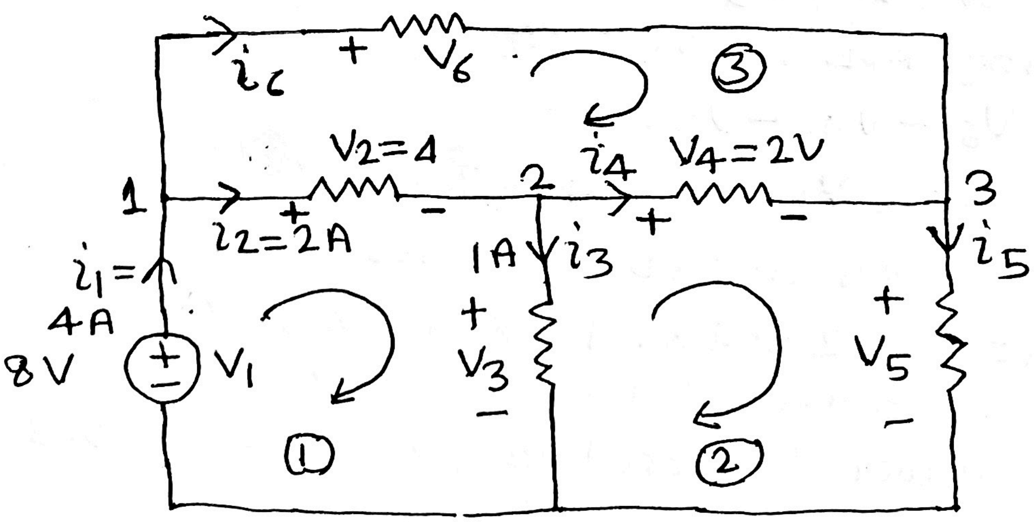
where $v_K(t)$ = Voltage across branch K

$i_K(t)$ = Current through branch K

Problem - For the following circuit, verify the Tellegen's theorem.



Solution -



Power delivered by the source

$$P_d = V_1 i_1 = 8 \times 4 = 32\,W$$

Power absorbed by resistors

$$P_a = V_2 i_2 + V_3 i_3 + V_4 i_4 + V_5 i_5 + V_6 i_6 \quad \text{--- (1)}$$

$i_2 = 2A$ $\quad i_4 = ?$ $\quad i_6 = ?$

$i_3 = 1A$ $\quad i_5 = ?$

$V_2 = 4V$ $\quad V_4 = 2V$ $\quad V_6 = ?$

$V_3 = ?$ $\quad V_5 = ?$

Using KCL at node 1

$i_1 = i_2 + i_6$

$4 = 2 + i_6 \Rightarrow i_6 = 4 - 2 = 2A$

KCL at node 2,

$i_2 = i_3 + i_4$

$2 = 1 + i_4 \Rightarrow i_4 = 2 - 1 = 1A$

KCL at node 3,

$i_5 = i_4 + i_6 = 1 + 2 = 3A$

Using KVL in mesh-1,

$V_2 + V_3 - V_1 = 0$

or $4 + V_3 - 8 = 0 \Rightarrow V_3 = 4V$

Using KVL in mesh-2,

$V_4 + V_5 - V_3 = 0$

or $2 + V_5 - 4 = 0 \Rightarrow V_5 = 2V$

Using KVL in mesh-3,

$V_6 - V_4 - V_2 = 0$

$V_6 = V_2 + V_4 = 4 + 2 = 6V$

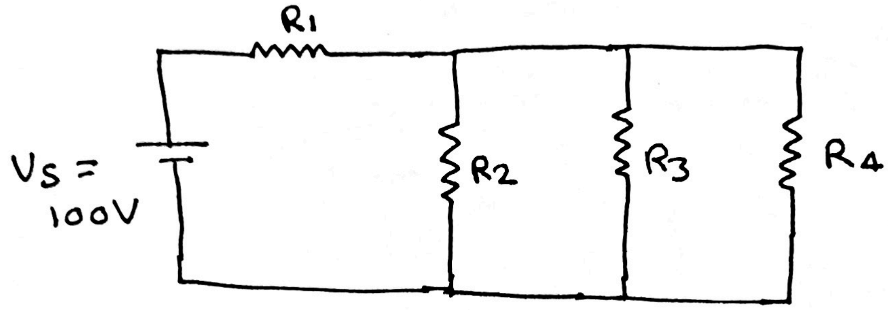
Hence Power absorbed by resistors

$P_a = 4 \times 2 + 4 \times 1 + 2 \times 1 + 2 \times 3 + 6 \times 2$

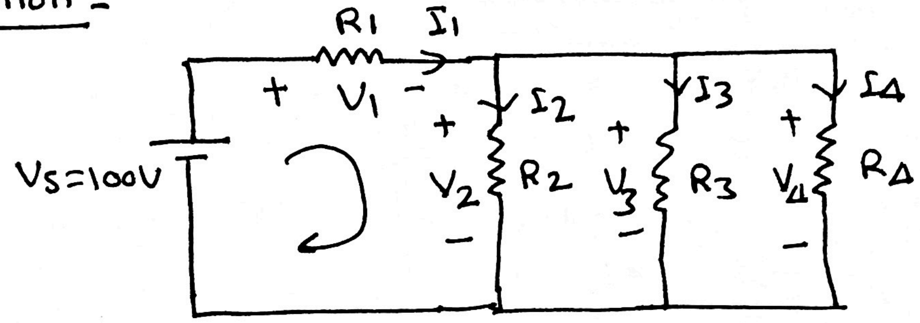
$= 8 + 4 + 2 + 6 + 12 = 32\,W$

Since Power delivered = Power absorbed, hence Tellegen's theorem is verified.

Example verify Tellegen's theorem for the circuit shown in fig- $[R_1 = 1\Omega, R_2 = R_3 = R_4 = 2\Omega]$



Solution -



Total Resistance across V_s, $R_T = R_1 + (R_2 || R_3 || R_4)$

$$R_T = 1 + (2||2||2) = 1 + \frac{2}{3} = \frac{5}{3}\ \Omega$$

$$I_1 = \frac{V_s}{R_T} = \frac{100}{5/3} = 60\ A$$

$$V_1 = I_1 \times R_1 = 60 \times 1 = 60\ V$$

By KVL, $V_1 + V_2 - 100 = 0$

$$60 + V_2 = 100$$

$$V_2 = 40\ V$$

$$V_3 = V_4 = V_2 = 40\ V$$

$$I_2 = \frac{V_2}{R_2} = \frac{40}{2} = 20\ A.$$

$$I_3 = I_4 = \frac{40}{2} = 20\ A$$

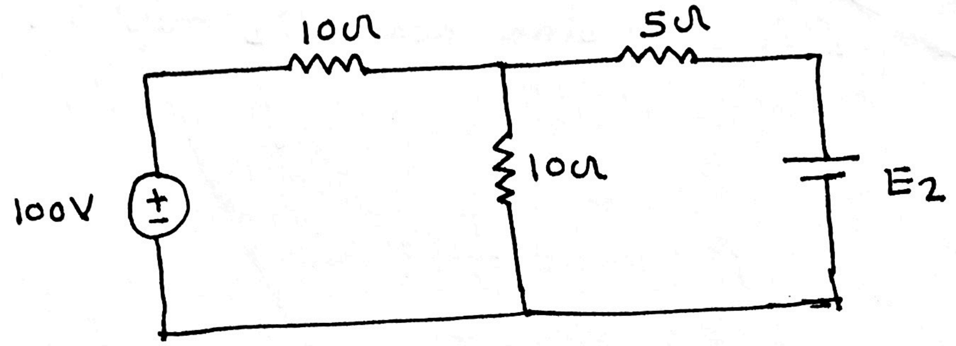
Power delevered by source $= V_s I_1 = 100 \times 60 = 6000\ W$

Power Absorbed by resistors $= V_1 I_1 + V_2 I_2 + V_3 I_3 + V_4 I_4$

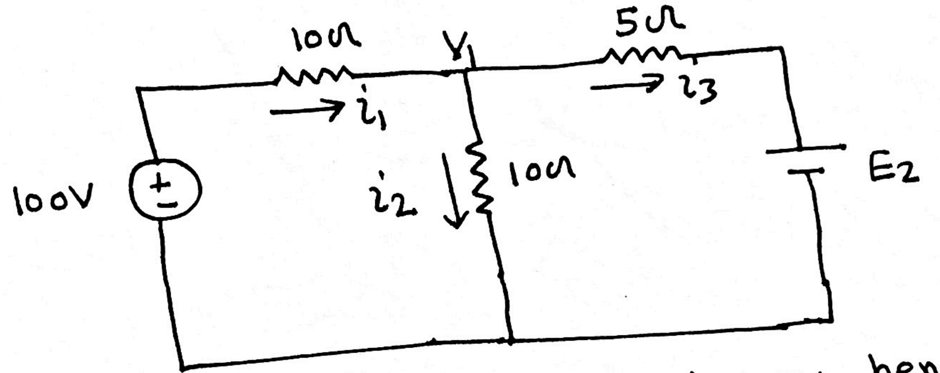
$= 60 \times 60 + 40 \times 20 + 40 \times 20 + 40 \times 20 = 6000\ W$

Since Power delivered = Power absorbed, hence Tellegen's Theorem is verified

Example Find the value of source E_2 in fig – using Tellegen's theorem, if the power absorbed by E_2 is 20 W.



Solution -



Since source E_2 is absorbing 20 W power hence

$$20 = i_3 \times E_2$$

$$i_3 = 20/E_2$$

From circuit, $i_3 = i_1 \times \frac{10}{5+10}$

$$\frac{20}{E_2} = i_1 \times \frac{10}{15}$$

or $i_1 = \frac{20 \times 15}{10 E_2} = \frac{30}{E_2}$

and $i_2 = i_1 \times \frac{5}{5+10} = \frac{30}{E_2} \times \frac{5}{15} = \frac{10}{E_2}$

According to Tellegen's theorem,

Power supplied by source = Power absorbed by other elements

$$100\, I_1 = I_1^2 \times 10 + I_2^2 \times 10 + I_3^2 \times 5 + 20$$

$$100 \times \frac{30}{E_2} = \left(\frac{30}{E_2}\right)^2 \times 10 + \left(\frac{10}{E_2}\right)^2 \times 10 + \left(\frac{20}{E_2}\right)^2 \times 5 + 20$$

or $E_2^2 - 150 E_2 + 600 = 0$

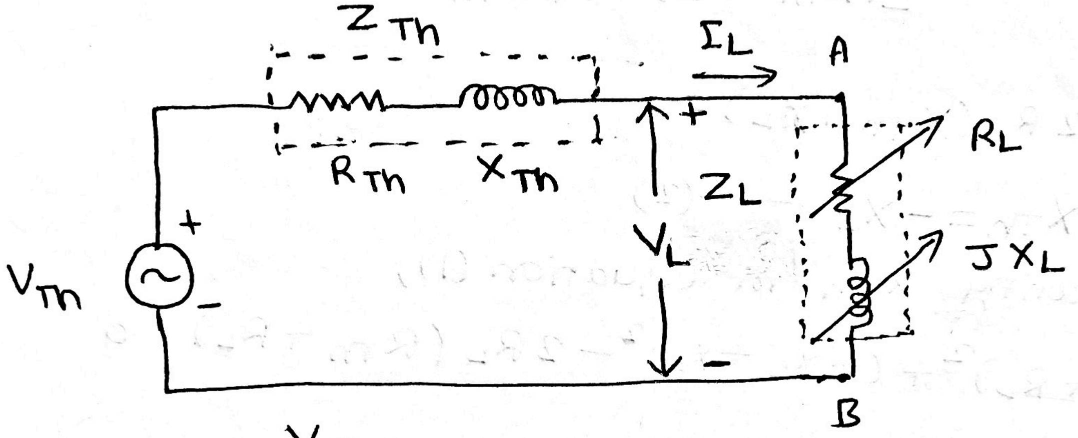
Solving it, we get

$$E_2 = 145.88V \quad ~~and~~ \quad or$$

$$E_2 = 4.113V$$

Since E_2 is absorbing hence E_2 must be 4.113 V

Maximum Power Transfer Theorem – This theorem states that " In an AC circuit, maximum power transfer occurs when its load impedance Z_L is equal to the conjugate of the thevenin's equivalent impedance" Hence $Z_L = Z_{Th}^*$



$$I_L = \frac{V_{Th}}{Z_{Th} + Z_L}$$

$$= \frac{V_{Th}}{(R_{Th} + JX_{Th}) + (R_L + JX_L)}$$

$$= \frac{V_{Th}}{(R_{Th} + R_L) + J(X_{Th} + X_L)}$$

Magnitude of Load current,

$$|I_L| = \frac{|V_{Th}|}{\sqrt{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}}$$

Average Power dissipated in load resistor is,

$$P_L = I_L^2 R_L = \frac{|V_{Th}|^2 R_L}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$$

For maximum power Transfer $\frac{\partial P_L}{\partial R_L} = 0$

and $\frac{\partial P_L}{\partial X_L} = 0$

$$\frac{\partial P_L}{\partial R_L} = \frac{|V_{Th}|^2\left[(R_{Th}+R_L)^2+(X_{Th}+X_L)^2 - R_L \cdot 2(R_{Th}+R_L)\right]}{\left[(R_{Th}+R_L)^2+(X_{Th}+X_L)^2\right]^2} = 0$$

or $(R_{Th}+R_L)^2+(X_{Th}+X_L)^2 - 2R_L(R_{Th}+R_L) = 0$ —(1)

$$\frac{\partial P_L}{\partial X_L} = \frac{|V_{Th}|^2\left[\{(R_{Th}+R_L)^2+(X_{Th}+X_L)^2\}\times 0 - R_L\, 2(X_{Th}+X_L)\right]}{\left[(R_{Th}+R_L)^2+(X_{Th}+X_L)^2\right]^2} = 0$$

or $-2R_L(X_{Th}+X_L) = 0$

or $X_{Th} = -X_L$ —(2)

Substituting X_{Th} in equation (1),

$(R_{Th}+R_L)^2 + (-X_L+X_L)^2 - 2R_L(R_{Th}+R_L) = 0$

or $R_{Th}+R_L = 2R_L$

or $R_{Th} = R_L$ —(3)

Since $Z_L = R_L + jX_L$ and $Z_{Th} = R_{Th} + jX_{Th}$

hence $Z_L = R_{Th} - jX_{Th}$

or $Z_L = Z_{Th}^*$

Hence for maximum power transfer Load impedance should be equal to the conjugate of Thevenin's equivalent impedance Z_{Th}.

* Load current $|I_L| = \dfrac{|V_{Th}|}{\sqrt{(R_{Th}+R_L)^2+(X_{Th}+X_L)^2}}$

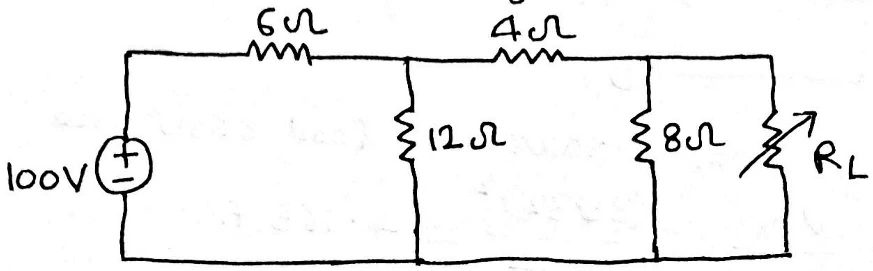
$$= \frac{|V_{Th}|}{\sqrt{(2R_L)^2+(-X_L+X_L)^2}}$$

$$= \frac{|V_{Th}|}{2R_L}$$

* $P_{L\,max} = \dfrac{|V_{Th}|^2 R_{Th}}{(R_{Th}+R_{Th})^2+(-X_L+X_L)^2} = \dfrac{|V_{Th}|^2 R_{Th}}{4R_{Th}}$

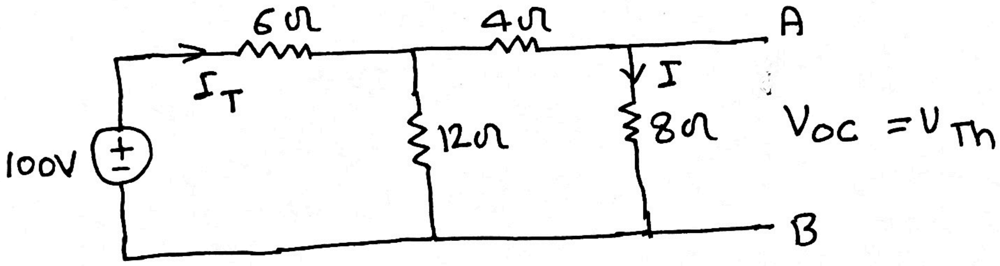
$$= \frac{|V_{Th}|^2 R_L}{4R_L}$$

Problem - Find the maximum power delivered to the load in the following circuit.



Solution - To determine the maximum power delivered to the load, first we have to determine the Thevenin equivalent circuit across the load.

Step-1 Determination of V_{Th}.



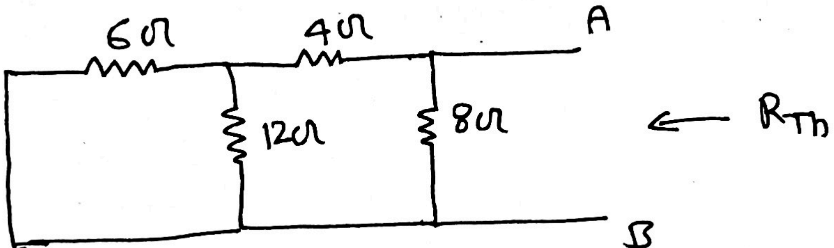
$$R_T = 6 + 12 \| (4+8)$$
$$= 6 + 12\|12 = 12\Omega$$

$$I_T = \frac{100}{R_T} = \frac{100}{12} = 8.33\,A$$

$$I = I_T \times \frac{12}{12+4+8} = 8.33 \times \frac{12}{24} = 4.165\,A$$

$$V_{oc} = I \times 8 = 4.165 \times 8 = 33.32\,V$$

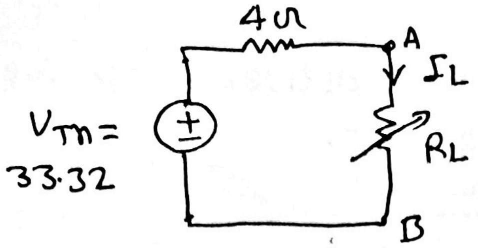
Step-2 Determination of R_{Th}



$$R_{Th} = [(6\|12) + 4]\|8 = \left[\frac{6\times12}{6+12} + 4\right]\|8$$
$$= 8\|8 = 4\Omega$$

Step-3 Determination of maximum power to the load

For maximum power Transfer $R_L = R_{Th} = 4\Omega$



Therefore current drawn by load resistance

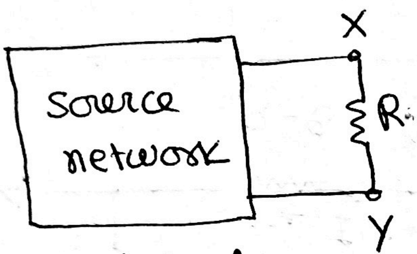
$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{33.32}{4+4} = 4.165\ A$$

Power delivered to the load $P_L = I_L^2 \times R_L$

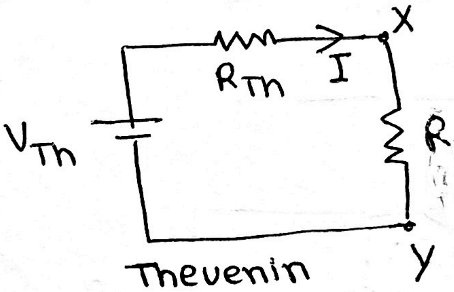
$$= (4.165)^2 \times 4$$

$$P_L = 69.39\ W$$

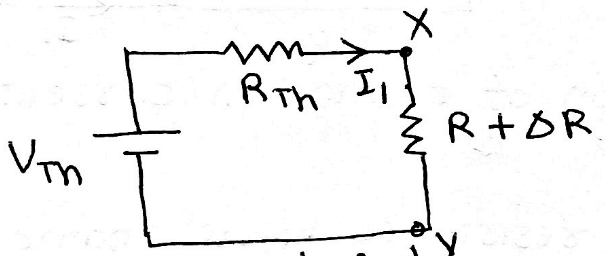
Compensation Theorem- The compensation theorem states that if the resistance R of any branch of a linear time invariant network is changed to $R+\Delta R$, the currents in all branches would changed, and these changes can be obtained by assuming that an ideal voltage source $I\Delta R$ has been connected in series with $R+\Delta R$, and all other sources in the network have been replaced by their internal resistances.



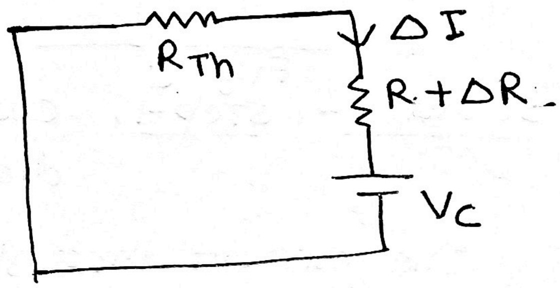
original network



Thevenin equivalent



Thevenin equivalent



Voltage source is replaced by its internal resistance

$$V_c = I\Delta R$$

$$\Delta I = \frac{-V_c}{R_{Th} + R + \Delta R}$$

* The compensating voltage V_c opposes the original current

Problem - In the circuit shown in fig-a the 6 Ω resistor is changed to 12 Ω. Find the current through the 4 Ω resistance before and after change in resistance. Determine the change in current ΔI through 4 Ω resistance using compensation theorem.

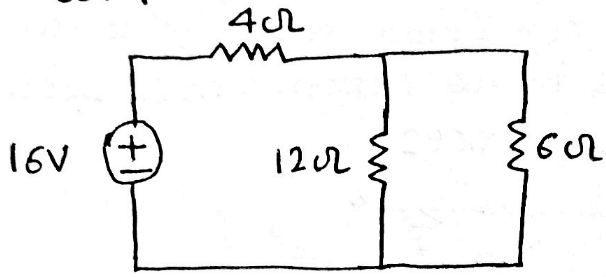


Fig-a

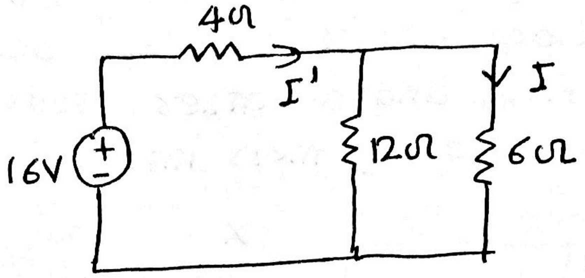


Fig-b

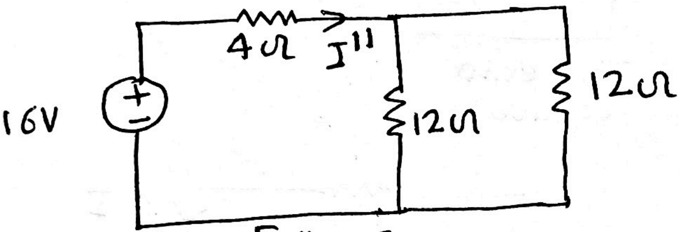


Fig-c

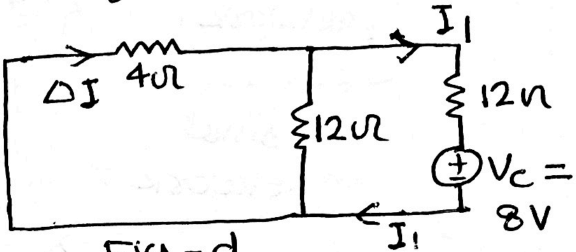


Fig-d

Solution- step-1, calculation of change in current directly.

The current through 4 Ω resistance before change

$$I' = \frac{16}{4+(12\|6)} = \frac{16}{4+4} = 2\ A$$

The current through 4 Ω resistance after change

$$I'' = \frac{16}{4+(12\|12)} = \frac{16}{10} = 1.6\ A$$

The change in current through 4 Ω resistor due to change in 6 Ω resistance,

$$\Delta I = I'' - I' = 1.6 - 2 = -0.4\,A$$

step-2 - calculation of change in current by compensation Theorem,

To determine ΔI using compensation theorem, replace 6 Ω by 12 Ω and connect a compensating voltage V_c in series with that and replace original 16 V source by a short circuit as shown in fig - d

The current in 6Ω resistance in original circuit

$$I = I' \times \frac{12}{12+6} = \frac{2 \times 12}{18} = 1.333\,A$$

compensating Voltage

$$V_c = I \Delta R = 1.333 \times (12-6)$$
$$= 7.998 \approx 8V$$

change in current through changed resistance

$$I_1 = -\frac{V_c}{12+(4 \| 12)} = -\frac{8}{12+3} = -\frac{8}{15} = -0.533\,A$$

Change in current through 4Ω resistance

$$\Delta I = I_1 \times \frac{12}{4+12} = -0.533 \times \frac{12}{16} = -0.4\,A$$

since ΔI is same in both the case, this proves the compensation theorem.